Science Faculty

Physics Department

Master Thesis

Properties Of Three Higgs Doublet Models and Dark Matter Candidates

خصائص النماذج التي تحتوي على ثلاث مزدوجات من حقل هيجز وجسيمات المادة المعتمة

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This Thesis was Submitted in Partial Fulfillment of the Requirements for the Master's Degree in Physics from the Faculty of Graduate Studies at Birzeit University, Palestine.

Thesis Committee:

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Abstract

The Standard Model (SM) of Particle Physics, which contains one Higgs Doublet only, has been tested experimentally for many decades and was successful in several aspects. But the Standard Model could not predict Dark Matter Candidates and Dark Energy, which means that it fails to describe **95**% of the universe. Models with Two Higgs Doublets (2HDMs) or Three Higgs Doublets (3HDMs) could accommodate Dark Matter Particles, these models are very attractive since they have rich spectrum of particles (Inert Scalar Particles) in addition to the particles that predicted by the Standard Model. In this thesis, we study a certain version of (3HDMs), and analyze a certain Real Vacuum Configuration called (R-I-1) of this model numerically. The results for the relic density we got is acceptable and falls within the allowed global range, when we studied two benchmark point of the mass of the lightest neutral inert scalar particle.

ملخص الرسالة

تم اختبار النموذج القياسي لفيزياء الجسيمات والذي يحتوي على مزدوجة هيجز واحدة فقط، تجريبيا لعدة عقود وكان ناجحا في العديد من الجوانب. لكن النموذج القياسي لم يستطع التنبؤ بجسيمات المادة المعتمة والطاقة المعتمة، مما يعني أنه فشل في وصف ٩٥ ٪ من الكون. وعليه تم دراسة نماذج أخرى تحتوي على اثنتين من مزدوجات هيجز والتي تتنبأ بجسيمات خاملة مرشحة لتكون جسيمات المادة المعتمة. في هذه الرسالة، سوف ندرس نموذج أخر يحتوي على ثلاث مزدوجات هيجز، مما يجعله غنيا جدا بالتنبؤ بجسيمات المادة المعتمة. الفكرة الأساسية في عملنا على هذه الرسالة هي أنه تم تحليل ودراسة نسخة معينة لأحد قيم تكوينات الفراغ الحقيقية لهذا النموذج، وكانت نتيجة التحليل العددي أننا حصلنا على قيمة مقبولة لكثافة البقايا المعتمدة عالميا. حيث قمنا بتحليل نقطتين من كتل الجسيم الخامل الذي نتج من هذا النموذج وذلك بعد تطبيق القيود النظرية والتجريبية الخاصة على هذا النموذج.

Acknowledgement

Praise be to God, whose grace good deeds are accomplished. Thanks be to God first and foremost for giving me the strength to achieve my dream of reaching the master's degree and writing a master's thesis. All thanks to my prestigious university, Birzeit, where I am very proud to be one of it undergraduate and master's students. All thanks and appreciation to all the university professors who taught me during my university period, especially the professors of the Faculty of Science, especially the professors of the Department of Physics. All thanks and appreciation to my wonderful supervisor, Dr. Wafaa Khater, who provided me with all the support and motivation in order to reach success and excellence. All thanks to those with whom I spent the best moments of my university life, to those whom I had the pleasure of knowing, my friends. I would also like to thank my loving mother, my dear father, my kind sister, my beloved brother. I would like to thank them all for their support in my life and throughout my study at the university.

Declaration

In this thesis, which I will present to a master's degree in Physics and then to the Department of Physics at Birzeit University, I confirm that this is my own research and my own work as well. This thesis is submitted for obtaining a master's degree only and not for obtaining a higher degree than that with all respect and appreciation.

Tarek Demadi

Signature : ....

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Chapter 1

Introduction

The **Standard Model (SM)** in particle physics is considered the basis for describing small scale physical particles that form the basic building blocks in the formation of the rest of the particles in nature. These building blocks are divided into two parts: **Bosons** and **Fermions**. It had also achieved many scientific achievements over the past decades, including the discovery of the Higgs Particle in **2012** [1–3]. All these achievements and discoveries make the Standard Model one of the greatest scientific models in the history of physics. Despite the huge achievements of the Standard Model, it could not explain many modern physical phenomena such as Dark Matter **(DM)** and its particles, in addition to that it did not contain many doublets of the Higgs Field, but only one Higgs Doublet existed inside it. This led to the emergence of many new physical scientific models that were considered as an extension of the Standard Model, such as **Two Higgs Doublet Model (2HDMs)** and **Three Higgs Doublet Model (3HDMs)**.

The subject of this thesis is focused on **Three Higgs Doublet Model (3HDMs)** which is considered one of the complex physical models extending from the Standard Model, as it predicts the existence of three doublets of the Higgs Field that may contain a large number of Higgs particles (perhaps the number of Higgs particles is above **8** particles) and therefore it could be one or more of these particles are candidates for Dark Matter particles. The potential part of the Lagrangian Equation of this model will be written and then addressing the **Real Vacuum Configurations** by using the **software micrOMEGA** [4, 5] in order to analyze the data and special equations for the one case of these configurations which was called **(R-I-1)** and then we make the required table (6.1) until the desired goal is reached in this thesis.

This thesis will be organized as follows: **Chapter 2** describes how this universe originated from the beginnings and how it accelerated, then it discusses the evidence for the existence of Dark Matter. **Chapter 3** talks about the Standard Model particles, in particular the Higgs particle and its characteristics, in addition to dealing with the topic of electroweak symmetry in the Standard Model. **Chapter 4** clarifies the simplest physical models extending from the Standard Model in the interpretation of Dark Matter particles, namely (2HDMs), in addition to clarifying the special case of (2HDMs), which is Inert Doublet Model (IDM) where one of the Higgs Doublets is inert (non active). **Chapter 5** describes the Three Higgs Doublet Model (3HDMs), which has a transformation under S_3 symmetry. **Chapter 6** analyze analytically and numerically a special case of Real Vacuum Configuration called **(R-I-1)** of this model and represent the results. **Finally Chapter 7** presents the conclusion. In the **Appendix (A)**, a review of Dark Matter in the String Theory is being presented.

Chapter 2

Evidence Of Dark Matter

2.1 Big Bang Cosmology

The Big Bang Cosmology consists of two parts which will be interpreted as follows:

- The Big Bang Nucleo Synthesis (BBN).
- The Cosmic Microwave Background (CMB).

The Big Bang Theory explained very vital things after the explosion, including that the universe had cooled and this allowed the formation of many subatomic particles and then the formation of atoms. Many important gases such as **Hydrogen (H)**, **Helium (He)** and other gases were formed and the attraction between them became very strong due to gravity, this also led to the formation of early stars and galaxies. After studying the subject of the Big Bang by many important astronomers, they noticed that there are gravitational effects in unknown regions around galaxies, which they later called it **Dark Matter or Dark Regions** [6], some of these effects were explained and others were not. The Scientific researches that concerned with the physics of the universe and galaxies are still continuing in the work of special studies in order to reveal the components of Dark Matter in this universe [7]. Many important studies confirming that the universe is expanding rapidly, so this is very important evidence for the existence of both Dark Matter and Dark Energy. Among these studies, the studies of the two scientists, **Georges Lemaitre and Edwin Hubble** who had great credit for revealing the expansion in the universe through their important studies that helped to reach an accurate detection of the components of the universe. The scientist Georges Lemaitre was considered the first one that notice in **(1927)** the universe came from only one point of origin which was called the **Primordial Atom** [8]. George Lemaitre described the evolution of the universe from its beginning from the primitive atom as follows:

- First: He suggested that all the mass of this universe exists in the form of a unique atom (Primordial Atom).
- Second: This universe will result from the disintegration of this atom and thus lead to a significant increase in the radius of the universe (where at first it was very small, but it wasn't even close to zero).
- Finally: This great expansion in the creation of the universe led to the formation of huge galaxies, stars and planets.

After that, George Lemaitre talked about **Cosmic Rays** and how they contributed greatly to the emergence of this universe. He suggested that these cosmic rays are fossils of the original Big Bang and they are similar to ordinary ash and smoke but differ from them in that they are very fast, this led to conclusion, that Galaxies and Stars formed millions of years ago without the need for an atmosphere, in addition, George also suggested that these Cosmic Rays are the basis for the formation of the universe from its original point which is the Primordial Atom [9]. **In 1931**, that is, four years after George Lemaître's theory of the origin of the universe from the point of origin which is the primordial atom. George developed this theory to become quantitative (The concept of the theory developed from the classical physics to modern physics), where George assumed that the universe is an **Elliptical Topological Curve** and this formal conception of the universe was able to explain that this universe expanded at the beginning and then went through a phase of stagnation until this expansion became very accelerated. Figure (2.1) shows how the Big Bang occurred and its time scale.

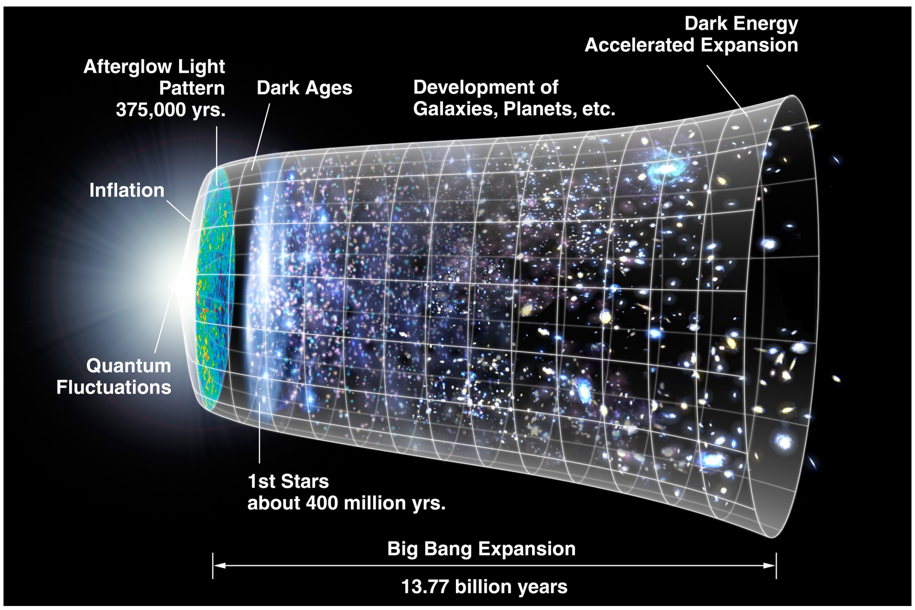

Figure 2.1: The Big Bang Cosmology [9].

Many physical quantities have been well studied and focused on, namely, the density of matter, the pressure of matter, as they change from time to another time, in addition to the existence of other than these quantities, there is a vital constant called **The Cosmological Constant (Λ)**. The scientist **Albert Einstein** was able to find this constant by solving some physical equations related to the physics of the universe using the necessary calculus methods. This cosmic constant has a close relationship with both Dark Matter and Dark Energy, as it is considered to represent the ratio between them, in addition to representing a constant energy density that fills the universe in a homogeneous manner. This constant has a fixed value, which will be explained as follows [9]:

$$\Lambda \;=\; 3\left(\frac{H_0}{c}\right)^2 \Omega_\Lambda \;= 1.1056\times 10^{-52}\; m^{-2} \tag{2.1}$$

- Ω_Λ = Lambda Contributions = 0.6889 ± 0.0056 . c = Speed of Light.
- H_0 = Hubble Constant = $(2.1927664 \pm 0.0136)\times 10^{-18}\; s^{-1}$.

2.2 The Cosmological Constant(Λ)

One of the most important achievements that have been reached from the theory of relativity is that the relationship between mass and energy has been determined, and this relationship has been reached by many physical equations, the most famous of which is **Einstein's Equation** which states by the following two statements:

- The mass is converted into energy and vice versa, and when this mass is converted, it gives very huge amounts of energy.
- The product of mass times the speed of light squared equals energy.

Einstein's equation has been called by name the **Mass Energy Equivalence Equation**, or more precisely, the **Matter Energy Quantization Principle**. This equation is given by the following relationship:

$$E = m\ c^2 \tag{2.2}$$

The vital cosmological constant **(**Λ**)** appeared in the **Einstein's Field Equation** where this equation is concerned with a field of physics called **Quantum Field Theory** and **Relativity Physics**. This Equation (2.3) gives a good explanation of the cosmological constant by linking it to some important physical quantities such as energy, momentum. This connection leads to a deep understanding of physical phenomena in the universe such as Dark Matter, Dark Energy and space - time. Equation (2.3) will be as follows [10]:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} \quad = \quad \frac{8\pi}{c^4}GT_{\mu\nu} \tag{2.3}$$

- Λ: Cosmological Constant = $1.1056 \times 10^{-52}\ m^{-2}$.
- $R_{\mu\nu}$: Ricci Curvature Tensor, used in differential geometry.
- R: Scalar Curvature.
- G: Universal Gravitational Constant = $6.674 \times 10^{-11}\ m^3\ .\ Kg^{-1}\ .\ s^{-2}$ [10].
- c: Speed of light = 2.99792458×10^8 m/s.
- $g_{\mu\nu}$: Metric Tensor, describe the structure of Space - Time
- $T_{\mu\nu}$: Stress Energy Tensor, describes the energy and momentum density in the Space - Time.

Summarizing the above, the important cosmological constant **(**Λ**)** is a term that was added by Einstein to his famous equation (2.3) in order to discover the equivalence between matter (Mass of Space) and vacuum energy, in addition to its importance in detecting both of Dark Matter and Dark Energy.

2.3 Lambda - Cold Dark Matter Model (ΛCDM)

The cosmological constant is the simplest possible explanation for Dark Energy in the universe, this constant is used only in cosmological physical models, the most famous is known as **(Lambda - Cold Dark Matter)**, it specializes in the study of cosmology and mass - energy density. This model is considered one of the simplest models that specializes in explaining the cosmic Big Bang, as it has a very great importance in explaining and simplifying the components of the universe in order to be properly understood. It consists of three important parts, which are as follows [11]:

- Lambda: The Cosmological Constant.
- The Cold Dark Matter and its abbreviation (CDM) with percentage **27**%.
- The Ordinary Matter and its abbreviation (OM) with percentage **5**%.

This model gives a good description and logical explanation for many important cosmic properties that would help in knowing the origin of the universe, how it is accelerating and the relationship of Dark Matter with its components. One more time, the name of this model has been recognized which is Lambda - Cold Dark Matter, but it will be explained in details as follows [11]:

- This model explained the definition of Dark Matter in a simple and easy way, as it was defined as a hypothetical substance that differs from ordinary matter such as protons, neutrons, electrons and neutrinos.
- Dark matter is given this name, because it does not appear to interact with an electromagnetic field, which means that it does not absorb, reflect or emit electromagnetic radiation, and therefore is difficult to detect.
- The meaning of the word "cold" in the name of this model is that the particles of Dark Matter move at a speed much less than the speed of light in vacuum.

Based on the previous percentages, the Dark Matter and the Dark Energy constitute **95**% of the total energy and mass content. The properties that have been studied by this model are too many, but some of them will be studied, and therefore they will be explained and presented as follows:

- First Property: Existence of the Cosmic Background in addition to its structure.
- Second Property: Great and accelerating expansion of this universe, which was observed through the light emanating from very distant galaxies, in addition to the vital presence of supernovae [12] .

Lambda - Cold Dark Matter Model explained the second property as follows [12]:

- This model includes an expansion related to space, such as:
  1. The Redshift that results from the absorption of a specific spectrum.
  2. The Lines of light emitted by these distant galaxies.
- The two effects mentioned in the previous point move in the vacuum in the form of electromagnetic waves.
- This expansion only increases the distance between objects (such as galaxies) but does not increase their size.
- This expansion also allows distant galaxies to move away from each other collectively at speeds exceeding the speed of light.

2.4 Big Bang Nucleo Synthesis (BBN)

The Big Bang Nucleo Synthesis (also known as primordial Nucleo Synthesis): It is the process of producing new nucleus with characteristics that differ from those of the light nucleus of isotopes of hydrogen, where the hydrogen - in addition to its isotopes - are the lightest elements in the universe because they contain a single proton in their nucleus. In general, **Isotopes** are defined as the atoms of the same element with equal **Atomic number** and differing in **Mass number** due to the different number of neutrons inside the nucleus. Most physicists specialized in cosmology believe that primitive Nucleo Synthesis occurred after the Big Bang in a short period of time not exceeding 20 minutes [13], In addition, they considered that it was responsible for the formation of many isotopes of some elements other than hydrogen, for example, isotopes of elements like Helium (He) ,Lithium (Li), Beryllium (Be) and other elements. Some of these isotopes formed due to (BBN) are as follows:

- Isotopes of Helium Element, which have a mass number of **3** or **4** with symbols He^3 , He^4.
- Isotope of Lithium Element, which has a mass number of **7** with symbol Li^7.
- Isotope of Beryllium Element, which has a mass number of **7** with symbol Be^7.

The process of creating light elements during (BBN) depends on many important parameters that provided a clear explanation of how the elements and their isotopes. These parameters will be displayed as follows [14]:

- First: The Neutron - Proton Ratio.
- Second: The Baryon - Photon Ratio.

The Neutron - Proton Ratio: This ratio calculated by the many nuclear interactions that occurred for a neutron with both positrons or electron neutrinos to produce protons and other products. The nuclear equations of this ratio will be presented as follows [14, 15]:

$$n + e^{+} \rightleftharpoons \bar{\nu}_e + p \tag{2.4}$$

$$n + \nu_e \rightleftharpoons p + e^{-} \tag{2.5}$$

- (n): Neutron. (p): Proton. (e^{+}) : Positron (Anti particle of Electron).
- (e^{-}): Electron. (ν_e): Electron Neutrino. ($\bar{\nu}_e$) : Anti Particle of Electron Neutrino.

The Baryon - Photon Ratio: This ratio is considered the vital basis for determining the abundance of light elements after the completion of the primitive Nucleo Synthesis process. it was also calculated within a set of nuclear equations whose most important products are the photon in addition to isotopes of some elements such as Hydrogen and Helium. These nuclear equations will be explained as follows [14, 15]:

$$p + n \rightarrow H^2 + \gamma \tag{2.6}$$

$$p + H^2 \rightarrow He^3 + \gamma \tag{2.7}$$

$$H^2 + H^2 \rightarrow He^3 + n \tag{2.8}$$

$$H^2 + H^2 \rightarrow H^3 + p \tag{2.9}$$

$$He^3 + H^2 \rightarrow He^4 + p \tag{2.10}$$

$$H^3 + H^2 \rightarrow He^4 + n \tag{2.11}$$

- (γ): Photon. (H^2): Isotope of Hydrogen - Deuterium. (H^3): Isotope of Hydrogen - Tritium.
- (He^3): Isotope of Helium. (He^4) : Isotope of Helium.

There are many observations about the previous nuclear equations, especially those related to the parameter of the **Baryon - Photon Ratio** which are as follows [14, 15]:

- First: The appearance of the photon due to these equations are incomplete and produce small amounts of both the Deuterium and the isotope helium (He^3). As a result, a photon must be present in order to conserve the total energy of these equations.

- Second: The relationship of this ratio with nuclear reactions is a direct relationship which will be clarified as follows:

  1. As the ratio increases, the reactions will increase, and this leads to an increase in the nuclei of helium isotopes.

  2. The lower the ratio, the reactions will be reduced, which leads to a decrease in the nuclei of helium isotopes.

All previous nuclear equations must be subject to the laws of **Conservation of Charge** and **Conservation of Mass** in order to write the reactants and products correctly and then obtain correct logical explanations for the reactions that occur. Each of them will be explained as follows:

- The Law of Conservation of Mass states that the sum of the masses of the reactants must equal the sum of the masses of the products.

- The Law of Conservation of Electric Charge states that the sum of the electric charges of the reactants must equal the sum of the electric charges of the products.

2.5 Friedman's Cosmological Equation

The Friedman equation for the universe is one of the most famous equations in describing both the universe and the energy of the universe. It is also important in revealing the existence of Dark Matter and Dark Energy as well. The Standard Model in particle physics depends mainly on the Friedman Equation, in order to explain many things, the most important of them are: how the total energy of the universe is distributed, in addition to explaining how this universe is expanding rapidly [16]. Friedman's Cosmological Equation will be as follows:

$$\sum_i \Omega_i = 1 + \left(\frac{K}{a(t).H}\right)^2 \tag{2.12}$$

- (K): Curvature Spatial Constant, it can be 1 or 0 or -1 For Fixed Distances.

- a(t): Scale Factor that multiplies Curvature Spatial Constant. (H): Hubble Parameter [17].

- ($\sum_i \Omega_i$): Total Energy Density Parameter. (Ω_i) : Ratio For Energy Density at time (t) of the (i) Cosmological Fluid.

The Friedman's Cosmological Equation worked to explain the total energy of the universe which is equal to the sum of all energies. These energies and their equations will be as follows [18]:

- Energy of All Particles (Protons, Neutrinos, Photons, Baryons ,....., etc).
- Dark Energy, Radiant Energy, Electromagnetic Energy ,....., etc.

$$\Omega_m = \Omega_b + \Omega_c \tag{2.13}$$

$$\Omega_{Total} = \Omega_m + \Omega_\Lambda + \Omega_{rad} + \Omega_\nu \tag{2.14}$$

$$\Omega_{Total} = \Omega_b + \Omega_c + \Omega_\Lambda + \Omega_{rad} + \Omega_\nu \tag{2.15}$$

- Ω_{Total}: Total Energy Density Parameter. Ω_m: Matter Density Parameter.
- Ω_b: Baryonic Density Parameter. Ω_c: CDM Density Parameter. Ω_Λ: Lambda Contributions.
- Ω_{rad}: Radiation Contributions. Ω_ν: Standard Model Neutrinos Contributions.

2.6 Cosmic Microwave Background (CMB)

The Cosmic Microwave Background (CMB) is an electromagnetic radiation that resulted from the remnants of an early stage of the formation of the universe, where (CMB) was called by another name, which is the remnant radiation [19], its considered a very important guide in identifying the origin of this universe and how stars, galaxies and radiation are formed, it was discovered by the American scientists **Arno Penzias** and **Robert Wilson** in **1965** [20]. Figure (2.2) shows the Cosmic Microwave Background in the universe.

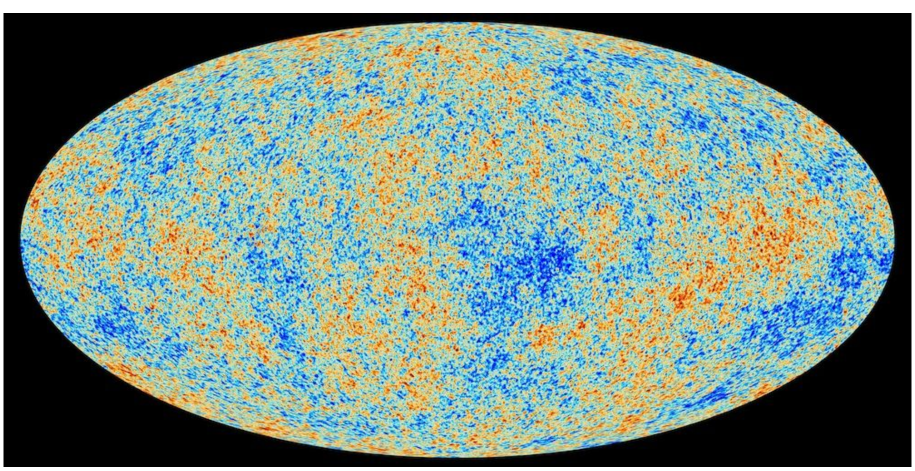

Figure 2.2: The Cosmic Microwave Background (CMB) [19, 20].

The Cosmic Microwave Background Radiation is also considered as a very important source of information and data about the reality of the universe, in addition to being considered the oldest source of electromagnetic radiation in this universe. The CMB Radiation is the origin of all radiation in this universe, among these vital radiation, **The Black Body Radiation**. One of the most important achievements of Black Body Radiation that is constituted a major qualitative leap from classical physics to modern physics. It is one of the most famous curves in modern physics which provided very important explanations that would help in the development of particle physics (particularly about the photon particle). Figure (2.3) shows the Black Body Radiation Curve [20].

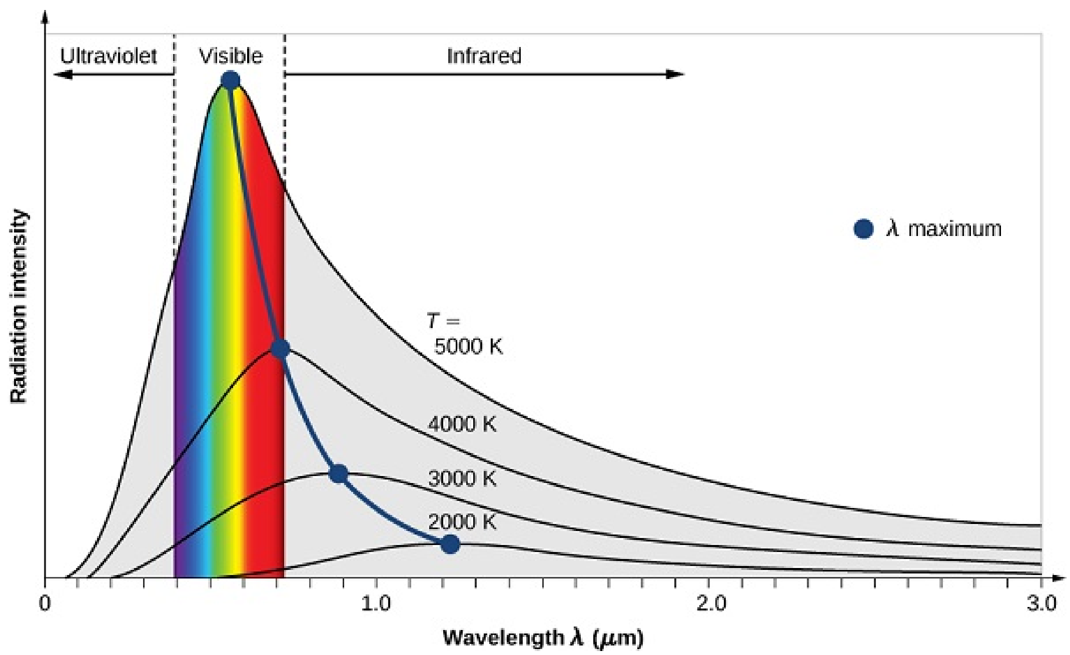

Figure 2.3: The Black Body Radiation (BBR) [20].

The mechanism of interpretation of the Black Body Radiation Curve will be explained as follows [20]:

1. The Radiation is emitted from a hot body as a result of the vibration of its atoms.

2. These atoms radiate specific beams called (photons), where the energy of each photon depends on the frequency of the atom from which it was issued.

3. At a certain temperature, the atoms do not vibrate with one frequency, but with different frequencies.

4. A few atoms vibrate at high frequencies (their wavelengths are short) and a few other atoms vibrate at low frequencies (their wavelengths are long), and most of the atoms have medium frequencies.

2.7 Relic Density For Energie's Densities From (Ω_{Total})

Once again, the equation for the total energy density parameter (Ω_{Total}) of the universe will be written again, in order to clarify its concepts more and more. It is as follows:

$$\Omega_m = \Omega_b + \Omega_c \tag{2.16}$$

$$\Omega_{Total} = \Omega_m + \Omega_\Lambda + \Omega_{rad} + \Omega_\nu \tag{2.17}$$

$$\Omega_{Total} = \Omega_b + \Omega_c + \Omega_\Lambda + \Omega_{rad} + \Omega_\nu \tag{2.18}$$

If the equation (2.16) is multiplied by the factor h^2, then it will be in the following form:

$$\Omega_m h^2 = \Omega_b h^2 + \Omega_c h^2 \tag{2.19}$$

- The values of the Relic Density of both **Matter Density Parameter** and **Cold Dark Matter Density Parameter** are being taken from reference [21]. They wil be as follows:

$$\Omega_m h^2 = 0.1430 \pm 0.0011 \tag{2.20}$$

$$\Omega_c h^2 = 0.1200 \pm 0.0012 \tag{2.21}$$

Where (h) is Planck's constant $= 6.62607004 \times 10^{-34}\ m^2.\ kg/s$

- The value of **Photon Density Parameter (Ω_{rad}) (The Radiation contributions)** will be explained as follows [21]:

$$\Omega_{rad} \simeq 2.47 \times 10^{-5}\ h^{-2} \tag{2.22}$$

- **The Neutrino Density Parameter (Ω_ν)** will be explained as follows [21]:

$$\Omega_\nu = \frac{\sum_i m_{v_i}}{93.14ev} \tag{2.23}$$

$$\sum_i m_{v_i} < 0.26eV \tag{2.24}$$

$$\Omega_\nu \leq 0.0279 \tag{2.25}$$

$\sum_i m_{v_i}$: The Sum Over The Neutrino Masses.

2.8 Dark Matter

It was previously talked about **Einstein's Equation** (2.2) about energy, which states that energy is converted into mass and also mass is converted into energy, but does this talk also apply to the relationship between Dark Matter (DM) and Dark Energy (DE) ?

- The answer to the previous question is as follows: **There are many differences between Dark Matter and Dark Energy, and therefore it is very difficult to say that one will transform into the other, as happens in ordinary matter with its energy.**

The Dark Matter is a hypothetical substance that differs from the known ordinary matter (such as protons, neutrons, electrons, neutrinos, ..., etc.). It does not seem to interact with an electromagnetic field, which means that it does not absorb, reflect or emit electromagnetic radiation, and therefore it is difficult to detect. [12]. Its constitutes **27**% of the components of the universe, unlike ordinary matter, which constitutes **5**% of the universe's components. It is treated as an **Attractive Force** that attracts the components of this universe, in other words, it slows down the expansion of the universe, it has been described as an attractive force because it interacts with gravity, and does not interact with the electromagnetic field [23]. Many physicists specializing in astrophysics and the universe have studied many equations of rotation for each galaxy, as it is expected that stars that are on the outer edges of the galaxy rotate at a slower speed than those that are very close to the center of the galaxy, as is the case in the rotation of planets in our solar system, but these scientists noticed that distant stars rotate at the same speed or slightly faster than those near, and this is evidence that there are invisible things working to hold this universe together, later called Dark Matter.

In Summary, the issue of discovering Dark Matter and its components is not an easy one. There are many scientific researches and cosmological models that have been and are continuing to the present time in order to detect Dark Matter, in addition to the use of many modern technologies for that purpose, but the required explanations have not been produced until now. Finally, the science continues to develop, as well as scientific research and models in order to obtain amazing and logical explanations about Dark Matter and its components.

2.9 Dark Energy

The Dark Energy is an unknown important type of energy that affects the universe in a very large proportion. It was first observed by supernovae, and it constitutes **68**% of the components of this universe. Its treated as a **Repulsive Force (Anti Gravity Force)**. In other words, it works to repel the components of the universe from each other, it makes this universe accelerate dramatically and does not make it accelerate at a constant rate [23]. Its also scattered everywhere in this universe, where it was previously talked that it constitutes a very large proportion of the components of the universe, this energy constantly increases its great influence as the universe accelerates, meaning that the relationship of Dark Energy with the acceleration of the universe is a **Direct Relationship**. In other words, the greater the expansion of the universe leads to a decrease in the force of gravity between the components of the universe. This leads to an increase in the dominance of Dark Energy in this universe, so, the relationship of Dark Energy with gravity is an **Inverse Relationship**.

Despite the low density of Dark Energy compared to Dark Matter and Ordinary Matter, it constituted a large proportion of the composition of space because its uniform across all space. Many theories have explained it, they show that there are many **Quantum Fluctuations** that formed a real source for accelerating the expansion of the universe and thus increasing the percentage of Dark Energy in this universe. Some of these theories talked about that there are many indications of the existence of Dark Energy, the most important of which is the long distances between galaxies that have to do with the **Redshift**, which greatly affected the expansion of the universe, thus increasing the percentage of Dark Energy. There are very important factors that play a fundamental role in the formation of Dark Energy in this universe and they are as follows [23]:

1. The Cosmological Constant, this constant constitutes a pivotal role in the physical equations of the physics of the universe, in addition to that it represents a constant energy density that fills this universe in a homogeneous manner.
2. The Standard (Scalar) Fields, which also play an important role in detecting Dark Energy, as they are dynamic quantities that have an energy density that is not fixed, unlike the cosmological constant, in other words, each of these standard fields has an energy density that varies in time and space.
3. The Cosmological Constant is very important in the cosmic equilibrium between matter, energy and gravity, as it was the beginning of the interest of many scientists, including Einstein, who talked about it in his Einstein's Field Equation (2.3). In addition to that, it is important in revealing the existence of both Dark Matter and Dark Energy [24].

Figure (2.4) shows the existence of both Dark Matter and Dark Energy in the universe.

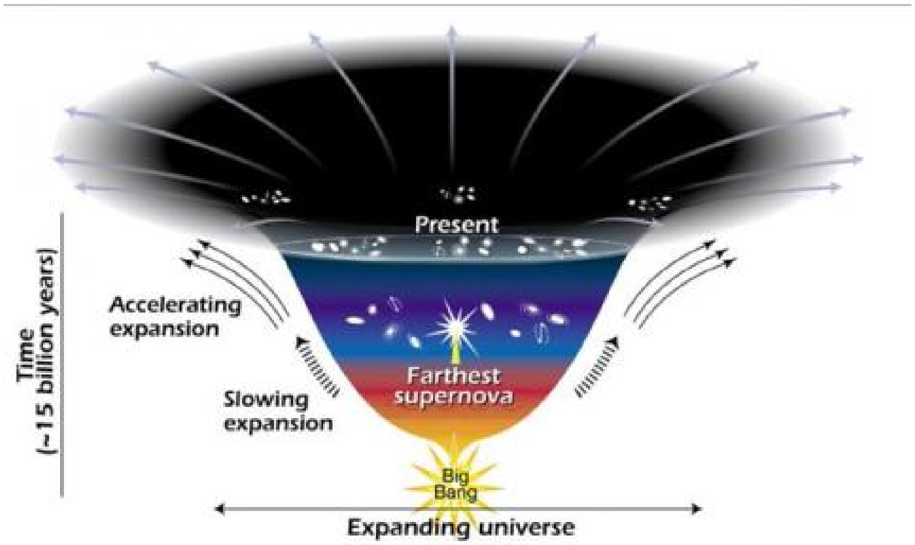

Figure 2.4: The Dark Matter Vs The Dark Energy [24].

2.10 Basic Evidence Of The Existence Of Dark Matter

In general, the law of general gravitation in physics is considered one of the most important physical laws that have a significant impact on cosmic physics. This law states that the force of attraction between two masses is directly proportional to the product of these two masses and inversely proportional to the square of the distance between them. The effect of this law appears clearly among very large masses, such as planets, while its effect on small masses, such as humans, is very weak. The mathematical formula for the law of universal gravitation will be as follows [10]:

$$F_g = \frac{G\ M_1\ M_2}{R^2} \tag{2.26}$$

- F_g: Force of Attraction between the Two Masses.
- G: Universal Gravitational Constant = $6.674 \times 10^{-11}\ m^3\ .\ Kg^{-1}\ .\ s^{-2}$.
- M_1: First Mass.
- M_2: Second Mass.
- R: Distance between the Two Masses.

The law of universal gravitation was discussed in terms of definition and mathematical formula in order to be linked with the very important evidence of the existence of Dark Matter, which is the **relationship between the velocity of the stars and their distance from the centers of their galaxies**. In general, galaxies **(Including Spiral Galaxies Ones)** contain objects close to their center and also objects far from their center. It is assumed that the mass of spiral galaxies is being concentrated in their centers and that objects near the center of galaxy rotate more quickly than objects far from the center [25]. In other words, the more objects are farther away from the centers of galaxies, the speed of their rotation will decrease due to the weakness of the gravitational force towards the center of the galaxy, similar to what is happening in our solar system. In this section, a very important physical evidence will be explained, as it is considered one of the most important scientific evidences for the existence of Dark Matter in the galaxies of our vast universe, and in particular our Milky Way galaxy [26].

Looking at the figure (2.5), the **Dashed Line** represents this assumption, but the **Continuous Line** is the correct actual line that explains the relationship of the velocity of stars in galaxies with their distance from the centers of their galaxies. With regard to the **Continuous Line**, its interpretation came through many physicists measuring the amount of light emitted from distant galaxies by means of specialized devices for that, then they represented the graphic relationship between the speeds of stars with their distance from the center of their galaxies. it was later found that the stars those far from the center of their galaxies rotate faster than expected and reverse the previous assumption [27].

So, scientists assumed the existence of a large unknown mass that is larger than the masses of galaxies that are not visible **(did not emit light)**, which was later called Dark Matter [28]. Figure (2.5) shows the relationship between the velocity of stars and their distance from the center of the galaxy.

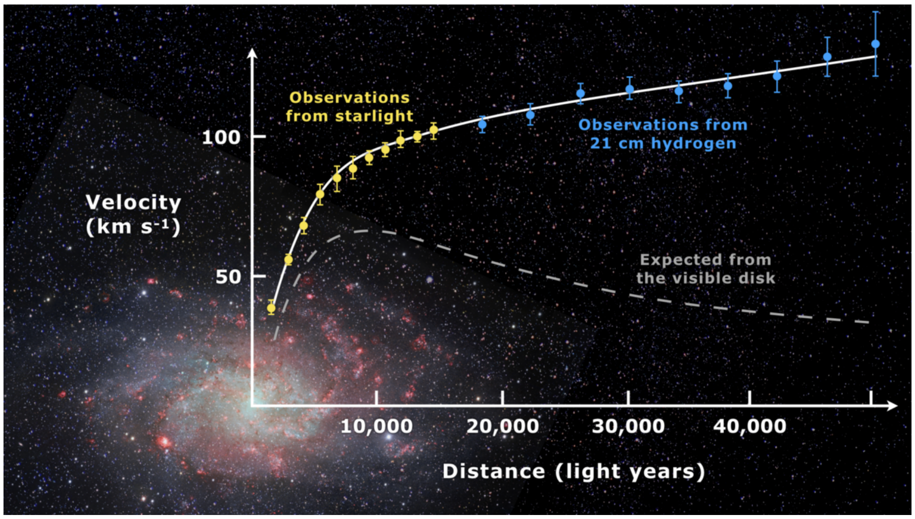

Figure 2.5: The Velocity of Stars Vs Their Distance From the Center of the Galaxy [25, 28].

2.11 Summary

Finally, we have reached the end of this chapter, where many interesting physics topics have been touched upon, such as the Cosmic Big Bang, the cosmic constant, cosmic microwave waves, black body radiation, Dark Matter, and finally Dark Energy. These topics were presented sequentially, starting from the beginning of the Big Bang and then up to the Standard Model. All of these previous topics that were explained were for one purpose, which is to prove the existence of Dark Matter and then the possibility of accessing its particles. Many well-known cosmic physical equations were also discussed, such as Einstein's Equation, Friedmann's Equation, in addition to nuclear equations on the subject of Big Bang Nucleo Synthesi. The properties of Dark Matter and Dark Energy were also explained, in addition to the effect of a percentage of each of them on the universe, where some important physical evidence of the existence of Dark Matter was reviewed. **In the next chapter,** we will discuss several important topics including the Standard Model, which constituted a very important quantum leap in particle physics as it is the comprehensive model for basic physical particles (Bosons and Fermions), then we will get to know one of its important particles, which is the Higgs Boson, in addition to identifying each of the physical symmetry groups, the transformation processes under these groups, Electroweak Symmetry and finally Electroweak Symmetry Breaking.

Chapter 3

Standard Model and Higgs Boson

3.1 The Standard Model

The Standard Model (SM) in Particle Physics is the theory that explains fundamental forces and fundamental particles in nature which are the electromagnetic forces, the strong and weak interaction forces.These forces are not related to the forces of gravity [29]. The Standard Model consists of two types of elementary particle that constitute the basic building block in the formation of all particles in this universe, as they will be explained as follows:

1. **Fermions**: The number of these particles is **12** where they were divided into two groups, each group includes **6** particles, these two groups are as follows [29]:

   a. **Quarks**: It is a type of fundamental particle that is included in the composition of the particles called **Hadrons**, such as protons and neutrons. They are named as: up (**u**) , down (**d**), top (**t**) , bottom (**b**) , charm (**c**) , strange (**s**).

   b. **Leptons**: It is another type of fundamental particle that makes up other particles, as it does not undergo strong interactions. They are named as: tau (τ) , tau neutrino (ν_τ) , muon (μ) , muon nutrino (ν_μ) , electron (**e**) , electron nutrino (ν_e).

2. **Bosons**: Their number according to the Standard Model is **5** particles, which were also divided into two groups, which are as follows [30]:

   a. Vector Bosons (Gauge Bosons): Which are **Gluons**, **Photons**, **W Bosons** and **Z Bosons**. These bosons have a spin.

   b. Scalar Bosons: Which is only contain one element that is the **Nutral Higgs Boson**. This boson has no spin.

It was said that the basic interactions that occur between elementary particles by strong, weak and electromagnetic forces are formulated in the Standard Model with the exception of gravity [30]. As a result, it was necessary to develop a special equation for the standard model that collects these forces in groups, where this equation will be in the following form:

$$G_{SM} = SU(3)\ X\ SU(2)\ X\ U(1) \tag{3.1}$$

The symbols for the equation (3.1) will be explained as the following:

- SU(3): It represents a Special Unitary Symmetry Group for Strong Interactions Forces.
- SU(2): It represents a Special Unitary Symmetry Group for Weak Interactions Forces.
- U(1): It represents a Unitary Symmetry Group for Electromagnetic Interactions Forces.

Figure(3.1) shows the Elementary Particles of the Standard Model:

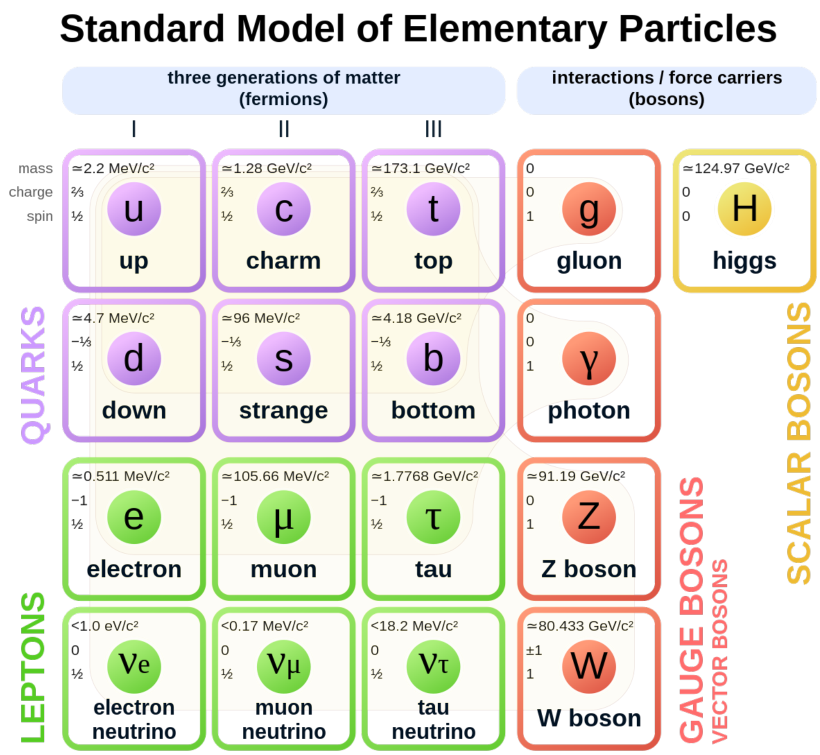

Figure 3.1: Elementary Particles of the Standard Model [30].

3.2 Higgs Boson Of The Standard Model

The Higgs Boson particle is considered one of the important particles in the Standard Model that constituted a qualitative leap in particle physics in particular and physics in general. Its existence has been predicted since the sixties of the last century through a group of scientific experiments called the **Higgs Mechanism**, and after a strenuous effort in working at the Large Hadron Collider by two physicists, **Peter Higgs** and **Francois Englert**, this particle was discovered and its properties were identified [2]. These two scientists were awarded the **2013 Nobel Prize** in physics for their active contribution to detecting the Higgs Boson and then introducing it to the Standard Model. The researches and scientific reports began focusing all their scientific content on the Higgs Boson particle in order to discover new physical particles [3].

- **Properties of the Standard Model's Higgs Boson**

There are many properties that have been discovered about the Higgs Boson, which are as follows [31]:

1. It has a spin = zero.
2. It has no electrical charge.
3. It results from quantum radiation in Higgs Fields.
4. It has no chromatic charge, in addition to being an almost Unstable particle that instantly decays into other particles.
5. It also helped in the detection of **Heavy Fermions** and **Gauge Bosons** which were obtained through the results of many important experiments in the Large Hadron Collider **(LHC)**.

There is also a very important topic to know in addition to the properties of the Higgs, where this topic will also be addressed regarding the Higgs particle, **which is its mass**. Its estimated at approximately **125 Gev/$\mathbf{c^2}$**. Many of the experiments that were conducted in both Large Hadron Collider **(LHC)** and ATLAS Detector established a specific mass scale for the Higgs mass between **115 Gev/$\mathbf{c^2}$** and **180 Gev/$\mathbf{c^2}$**. The reason for not reaching a fixed value for the Higgs mass is its close association with other bosons (W Boson, Z Boson) through electroweak symmetry, in addition to the **Hierarchy Problem** [32]. The properties of the Higgs Boson were discussed, this also requires the interpretation of the Higgs Field, it means that a group of Higgs particles interact with each other in a series of interactions, its a scalar (non vector) field that does not have a spin like other Bosons Fields, it consists of four components, two of them have no electric charge (Neutral Components), while the other two have an electric charge [33]. One of the important things that must be studied about the Higgs Field is that it consumes little energy in its ground state until a certain vacuum expectation value **(not equal to zero)** is obtained. This value is very important as it plays a necessary role in breaking the electroweak symmetry with the help of a specific mechanism called the Higgs Mechanism. The goal of this mechanism, with the existence of this value, is to give certain masses to the rest of the bosons in the Standard Model, namely W Boson and Z Boson [34]. There is also another important property of the Higgs Field, which is that its potential resembles a **Mexican Hat** see Fig (3.2) when it is graphed. The location of this hat expresses low energy ground state of its potential and the reason for the presence of this hat is the electroweak symmetry that is broken by the Higgs Mechanism [35]. Finally, after addressing the properties of each of the Higgs Boson and its field, the subject of Higgs Field Doublet will be discussed in the next section **(3.3)**. The Standard Model contains one doublet of the Higgs Field, but this doublet did not make the Standard Model able to detect new physical particles, which led to the emergence of extended physical models of the Standard Model that will be addressed later in this thesis [36].

3.3 Standard Model's Higgs Potential

With regard to the **Lagrangian Density Equation** which consists of two parts: **The Kinetic Term** and **The Potential Term**. The kinetic term is only concerned with describing the motion of particles and this term cannot reveal the presence of mass particles While the potential term is the term that responsible for the detection of mass particles. With regard to the important equation of **Higgs Potential** that related for the only one doublet for Higgs Field in the Standard Model will be in the following form [37]:

$$V(\Phi^{\dagger}\Phi) \;=\; \frac{\lambda}{4}\,(\Phi^{\dagger}\Phi)^2 \;+\; \mu^2\,(\Phi^{\dagger}\Phi)\;,\;\lambda > 0 \tag{3.2}$$

- Φ: Higgs Field Doublet.
- $\Phi^{\dagger}$: Hermitian Conjucate Of Higgs Field Doublet.
- λ: Real Coupling Constant that Describes Self Interactions Among Scalar Higgs Fields.
- μ: Real Coupling Constant that also Describes Self Interactions Among Scalar Higgs Fields.

Figure(3.2) Shows The Standard Model's Higgs Potential.

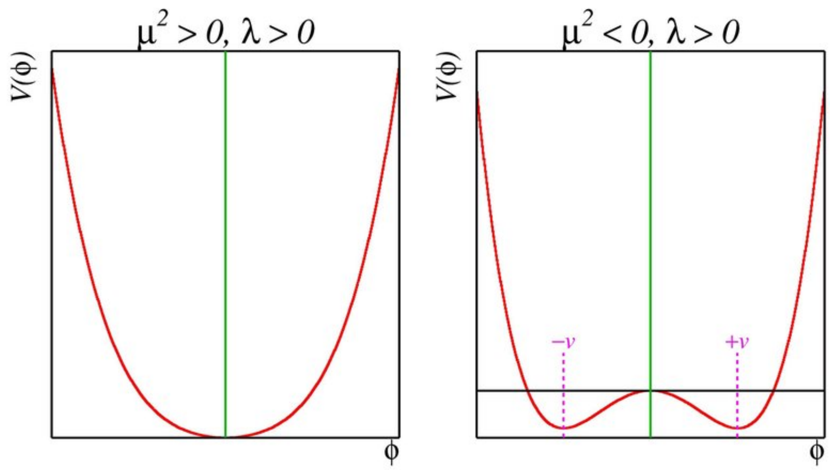

Figure 3.2: The Standard Model's Higgs Potential [34, 35].

3.4 The Electroweak Symmetry

In Physics, the **Symmetry** of a physical system is a physical or mathematical feature of the system observed or intrinsic that is maintained or remains unchanged under some transformation. Symmetries may be broadly classified as **Global** or **Local**, these two types of symmetry will be explained as follows:

- **Global Symmetry**: is one that maintains a constant property of a transformation that is applied simultaneously at all points of Space Time. For example, Lepton Number is a **U(1)** global symmetry that guarantees that the lightest lepton is stable.

- **Local Symmetry**: is one that keeps a property invariant when a possibly different symmetry transformation is applied at each point of Space Time.

There are four main physical important forces present in the nature and these are considered the most important forces of physics that had been studied intensively in the department of particle physics **(High Energy Physics)**. These four forces are electromagnetic forces, strong forces, weak forces and finally gravitational forces. The **Electroweak Symmetry** relates to the electroweak interactions that occur between only two of the four types of physical forces, namely the Electromagnetic Forces and the Weak Forces. Due to the names of these two types of forces, its name was combined to be Electroweak Symmetry [38]. This symmetry consists of two parts of the fields, the first part is called **Weak Isospin Field** produced by weak interactions, while the second part is called **Weak Hyper Charge Field** resulting from electromagnetic interactions. In Particle Physics and the Standard Model, the special unitary groups SU(n) were explained, but for the electroweak symmetry, the focus was only on each of two groups, the first one SU(2) symmetry group which is concerned with the weak interactions, the second one U(1) symmetry group which is concerned with the electromagnetic interactions. Many scientific researches in particle physics have been published that the main responsible for the electroweak interactions are the gauge bosons (W^+, W^-) and the neutral gauge boson (Z^0). As for the special unitary groups SU(n), there is a very important thing to know, which is the number of generators for each group of them, where the number of generators refers to the number of elements within each of these special unitary groups. As a result, the equation for the number of generators will be clarified as follows [38]:

$$The\ Number\ of\ generators\ for\ SU(n)\ groups\ =\ n^2\ -\ 1 \tag{3.3}$$

Once again, the electroweak symmetry represents a very important symmetry in reaching necessary physical explanations related to the Standard Model, as this symmetry is a combination of two of the four types of physical forces, which are the weak forces and the electromagnetic forces. This symmetry is related to many important physical processes such as the transformation under symmetry groups in the Standard Model which are written as the following **($SU(3)$ x $SU(2)$ x $U(1)_Y$)**. In some cases the transformation will be from groups **($SU(2)$ x $U(1)_Y$)** to group $U(1)_{EM}$ and this is known as the **Electroweak Symmetry Breaking**. it is necessary to review the generators for each of the symmetry groups of the Standard Model, as all these generators have a special name and a special symbol as well. All of them will be explained as follows [39]:

- The SU(3) has **8** generators, called Gluon Fields with symbol (G^a_μ).

- The SU(2) has **3** generators, called Gauge Fields with symbols $(W^1_\mu, W^2_\mu, W^3_\mu)$.

- The U(1) has only **1** generator, called Gauge Field with symbol (B_μ).

As for the electroweak symmetry, it was previously said that this symmetry is found within the special unitary groups ($SU(2) \times U(1)$). Each group of them has its own generators, the first one is called weak isospin, its considered a generator of a special unitary group $SU(2)$ in addition to being denoted by the symbol (T) or (T_3), while the second one is called weak hyper charge, its considered a generator of the unitary group $U(1)$ and it is denoted by the following symbol (Y_w). These two generators are considered to be the basis for giving the value of the charge for all particles in the Standard Model. They are originally two quantum numbers associated with the charge operator (Q) through an important equation that will be clarified as follows [39]:

$$Q \;=\; T_3 \;+\; \frac{1}{2}\, Y_w \tag{3.4}$$

- Q: It represents the actual value of the charge for each Standard Model's particles.
- $\mathbf{T_3}$: This symbol represents a quantum number called Weak Isospin.
- $\mathbf{Y_w}$: This symbol represents a quantum number called Weak Hyper Charge.

The electroweak symmetry is one of the important symmetries in particle physics, and in particular it plays an important role in the subject of the Higgs Field. Its importance stems from the fact that it combines two main types of the four physical forces as we talked previously. Therefore, it has become very important to study the **Bosonic Higgs Lagrangian Equation** for this vital symmetry due to its extreme importance in adding valuable scientific information about electroweak symmetry. This equation will be clarified as follows [39]:

$$\begin{aligned} \ell_{Bosonic} \;=\; & \mid D_\mu \Phi(x) \mid^2 \;-\; \mu^2 \mid \Phi(x) \mid^2 \;-\; \lambda \mid \Phi(x) \mid^4 \\ & -\; \frac{1}{4}\, B_{\mu\nu} B^{\mu\nu} \;-\; \frac{1}{4}\, W^a_{\mu\nu} W^{a,\mu\nu} \end{aligned} \tag{3.5}$$

- $\Phi(x)$: It represents Higgs Field.
- μ, λ: They represent Real Coupling Constants.
- $D_\mu, B_{\mu\nu}, W^a_{\mu\nu}$: They represent Covariant Derivatives.

$$D_\mu \;=\; \partial_\mu \;+\; ig\, \frac{\tau^a}{2}\, W^a_\mu \;+\; i\acute{g}\, \frac{Y}{2}\, B_\mu \tag{3.6}$$

$$B_{\mu\nu} \;=\; \partial_\mu B_\nu \;-\; \partial_\nu B_\mu \tag{3.7}$$

$$W^a_{\mu\nu} \;=\; \partial_\mu W^a_\nu \;-\; \partial_\nu W^a_\mu \;-\; g f^{abc}\, W^b_\mu\, W^c_\nu \tag{3.8}$$

As for the equation (3.5) of electroweak symmetry, there is an important symbol in it, which is the Higgs Field $\Phi(x)$. This field has a matrix mathematical formula consisting of two complex scalar fields. This formula will be explained as follows [40]:

$$\Phi(x) \;=\; \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_1(x) + i\eta_2(x) \\ \nu + \sigma(x) + i\eta_3(x) \end{pmatrix} \tag{3.9}$$

- $\eta_1(x)$, $\eta_2(x)$, $\eta_3(x)$, $\sigma(x)$: They represent Real Fields.
- ν: Standard Model's Vacuum Expectation Value $\approx$ 246 Gev.

There is a very important thing to focus on in studying both the Standard Model and all kinds of symmetries in Particle Physics, which is that the **Lagrangian Density Equation** for physical fields. It must be remain invariant under all transformations of the three symmetry groups **(**$SU(3)$ **x** $SU(2)$ **x** $U(1)_Y$**)** that make up the Standard Model. However, there are some particles in the Standard Model, such as fermions with both quarks and leptons transforming only under the special unitary group $SU(3)$. In order to carry out the total transformation process for any of the physical fields, this require transformation equation to achieve this purpose in addition to taking into account the equation of total Covariant Derivative (D_μ). These equations will be explained by the following [40]:

$$\Psi_i(x) \;\rightarrow\; exp\left(i\alpha_1(x) g_1 \frac{Y}{2} \;+\; i\alpha_2^l(x) g_2 \frac{\sigma_l}{2} P_L \;+\; i\alpha_3^a(x) g_3 \frac{\lambda_a}{2} \right) \Psi_i(x) \tag{3.10}$$

$$D_\mu \;=\; \partial_\mu \;-\; i g_1 \frac{Y}{2} B_\mu(x) \;-\; i g_2 W_\mu^l \frac{\sigma_l}{2} P_L \;-\; i g_3 G_\mu^a(x) \frac{\lambda_a}{2} \tag{3.11}$$

- $\alpha_{1,2,3}(x)$: They represent the transformation parameters.
- g_1, g_2, g_3: They associated with three factors of the Standard Model gauge group.
- $g_1 \frac{Y}{2}$: This Combination play the role of the charge q for the $U(1)$ symmetry.
- $\frac{\sigma_l}{2}$: This generator plays an important role in the $SU(2)$ transformations.
- $\frac{\lambda_a}{2}$: This generator plays an important role in the $SU(3)$ transformations.
- P_L: The Projection Operator and this ensures that $SU(2)$ transformations only act on Left Chiral Fermions, while leaving Right Chiral Fermions unchanged.

The transformation equations for any physical field in particle physics under each group of symmetry groups SU(3), SU(2), U(1) respectively will be derived from the original basic equation (3.10). These equations will be explained as follows [40]:

$$\left[\Psi_i(x) \;\longrightarrow\; exp\left(\; i\alpha_3^a(x) g_3 \frac{\lambda_a}{2}\;\right) \Psi_i(x)\right] \;\longrightarrow\; Under\ SU(3)\ Symmetry\ group \tag{3.12}$$

$$\left[\Psi_i(x) \;\rightarrow\; exp\left(\; i\alpha_2^l(x) g_2 \frac{\sigma_l}{2} P_L \;\right) \Psi_i(x)\right] \;\longrightarrow\; Under\ SU(2)\ Symmetry\ group \tag{3.13}$$

$$\left[\Psi_i(x) \;\longrightarrow\; exp\left(\; i\alpha_1(x) g_1 \frac{Y}{2}\;\right) \Psi_i(x)\right] \;\longrightarrow\; Under\ U(1)\ Symmetry\ group \tag{3.14}$$

With regard to the Higgs Field $\Phi(x)$, it transforms under all the previous symmetry groups of the Standard Model, except the symmetry group $SU(3)$, due to its physical properties that differ from the rest of the Standard Model particles as it is considered a scalar particle. In addition to that there is **No notation for Chirality** on the scalars like Higgs Boson, and as a result of these properties that distinguish the Higgs Boson from the rest of the particles, this would cancel both terms (λ_a) and (P_L) from the new important transformation equation for the Higgs Boson, so that, this equation will be explained as follows [41]:

$$\Phi_i(x) \;\rightarrow\; exp\left(\; i\alpha_1(x) g_1 \frac{Y}{2} \;+\; i\alpha_2^l(x) g_2 \frac{\sigma_l}{2}\;\right) \Phi_i(x) \tag{3.15}$$

After talking about the transformation equations under the three symmetry groups of the Standard Model, it must also talk about a very important equation, as it is considered one of the most important equations in particle physics, namely **Standard Model's Lagrangian Density Equation**. This equation explains all the details related to the interactions of particles of the Standard Model. It will be explained as follows [41]:

$$\begin{aligned} \ell_{SM} \;=\; & -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} \;-\; \frac{1}{4} W^l_{\mu\nu} W^{\mu\nu,l} \;-\; \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + \sum_i \bar{\psi}_i \; i \; \gamma^\mu \; D_\mu \; \psi_i \\ & + (D_\mu \Phi)^\dagger D_\mu \Phi \;-\; V(\Phi^\dagger \Phi) \;-\; \sum_{i,j} (\; y_{ij} \; \Phi \; \bar{\psi}_i \; \psi_j \;+\; h.c\;) \end{aligned} \tag{3.16}$$

- The First Line of Eq (3.16) represents the Kinetic terms for the gauge bosons.
- The Second Line of Eq (3.16) represents the vital Dirac Lagrangian that describing the Standard Model Fermions.
- The Third Line of Eq (3.16) represents the important Higgs Sector with $V(\Phi^\dagger\Phi)$ that defined in Eq (3.2).

$$G^a_{\mu\nu} \;=\; \partial_\mu G^a_\nu(x) \;-\; \partial_\nu G^a_\mu(x) \;-\; g_3\ f^a_{bc}\left[G^b_\mu(x), G^c_\nu(x)\right] \tag{3.17}$$

$$W^l_{\mu\nu} \;=\; \partial_\mu W^l_\nu(x) \;-\; \partial_\nu W^l_\mu(x) \;-\; g_2\ \epsilon^l_{mn}\left[W^m_\mu(x), W^n_\nu(x)\right] \tag{3.18}$$

$$F_{\mu\nu} \;=\; \partial_\mu B_\nu(x) \;-\; \partial_\nu B_\mu(x) \tag{3.19}$$

Regarding the second line of equation (3.16), this line talks about **Dirac Lagrangian**, which describes the fermion particles. This Lagrangian is invariant under the transformation of the symmetry group $SU(3)$ of the Standard Model. In this line also, it is necessary to focus on the terms (D_μ) and (γ^μ), where the first symbol represents a Covariant Derivative that depends on many physical fields in addition to many important constants, while the second symbol represents a **Quaternary Matrix** called **Weyl basis** or **Chiral basis** or **Gamma Matrices**. The equation of (γ^μ) will be written as follows [42]:

$$\gamma^\mu \;=\; \gamma^i \;=\; \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \tag{3.20}$$

- Where (σ^i) are called the **Pauli Matrices**, they are very important mathematical matrices of the order **(2x2)**. Each of the them will be illustrated as follows [42]:

$$\sigma^1 \;=\; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \sigma^2 \;=\; \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \sigma^3 \;=\; \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{3.21}$$

- After the **Three Pauli Matrices** were identified, it became possible to write the vital mathematical matrix formula for each of **Gamma Matrices** (γ^i), where the formula of each of them will be explained as follows [42]:

$$\gamma^1 \;=\; \begin{pmatrix} 0 & \sigma^1 \\ -\sigma^1 & 0 \end{pmatrix} \;=\; \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \tag{3.22}$$

$$\gamma^2 \;=\; \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix} \;=\; \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \tag{3.23}$$

$$\gamma^3 \;=\; \begin{pmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{pmatrix} \;=\; \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{3.24}$$

For the last third line of equation (3.16), this line represents the **Higgs Sector**, Where this section explains many important matters in particle physics **(High Energy Physics)**. It includes all the interactions that occur between the fields of the Higgs particles with gauge bosons, in addition to that this section explains the interactions that occur between the Higgs particle with particles of the Standard Model such as Fermions. Each term of this line will be interpreted as follows [43]:

- $(D_\mu \Phi)^\dagger D_\mu \Phi$: It represents the Kinetic term, as it contains the Covariant Derivative (D_μ) which is the basis for the interactions that occur between the Higgs Boson and the Gauge Bosons (Vector Bosons) [43].

- $V(\Phi^\dagger \Phi)$: It represents the Potential term, it contains all the interactions that occur between the Fields of the Higgs particles with each other only [43].

- $\sum_{i,j}(\ y_{ij}\ \Phi\ \bar{\psi}_i\ \psi_j\ +\ h.c\ )$: It represents the interactions that occur between the Higgs Bosons (Scalar Particles) with Standard Model Fermions (Spinors) [43].

- y_{ij}: It represents Coupling Constants.

3.5 The Symmetry Breaking Of The Higgs Potential

In general, with regard to the Higgs Potential $V(\Phi^\dagger \Phi)$ in equation (3.2), this term is considered one of the most important equations in **High Energy Physics**, as it is the basis for clarifying very important information about the process of symmetry breaking that can occur for the three symmetry groups of the Standard Model. The symmetry breaking is one of the very important physical topics that have a great impact in providing many necessary explanations for complex physical phenomena, as it is not considered one of the easy topics in High Energy Physics. The symmetry breaking in the primitive concept is that any physical system was in a state of symmetry and then became in a state of asymmetry as a result of **External Disturbances** or **External Interactions** [44].

The electroweak symmetry includes the merging of both the weak interactions and the electromagnetic interactions, which increases the difficulty of the possibility of breaking it [45]. The bosons $(W^\mp)$ and (Z^0) are closely related to electroweak symmetry due to electromagnetic interactions and weak interactions that occur between them [46]. The electroweak symmetry breaking plays an important role in many physical topics, the most important of which, is the issue of the mass of each particle of gauge bosons in the Standard Model especially $(W^\mp)$ and (Z^0). This symmetry breaking will not being happened without the help of the so called Higgs Mechanism, without this mechanism, these gauge bosons would become massless particles [47]. The basic idea of the Higgs Mechanism, in short, is to give masses to the measurement bosons $(W^\mp)$ and (Z^0), without this mechanism, it would be difficult to find masses for these bosons (become have no masses). During this break, the bosons that are formed in this case are called **Goldstone Bosons** and then after a series of interactions, these bosons become massive particles. This mechanism is related only to local symmetry and not to global symmetry. The reason for that, this mechanism differs from one symmetry group to another symmetry group (means SU(3), SU(2) and U(1)) [48].

In other words, it is difficult to make the Higgs Mechanism to be a single and comprehensive mechanism for all physical symmetry groups. The main reason for the difference in this mechanism is the difference in the Lagrange Density Equation for each symmetry group due to the difference in both the kinetic term and the potential term that consist the two main important terms in the original Lagrangian Density Equation.

3.6 Summary

Finally, we have reached the end of this chapter, where very important physics topics have been addressed, the most important of which is the subject of the Standard Model, which constitutes a real important achievement in particle physics in particular because it contains the necessary basic building blocks that include in the composition of the rest physical particles in the nature known as Bosons and Fermions. The Higgs particle, Higgs Field, Higgs Potential, Electroweak Symmetry, and finally the Electroweak Symmetry Breaking of the Higgs Potential were also discussed. We also saw how transformation processes occurred under the three physical symmetry groups through each of the physical matrices and equations specific to the transformation under these symmetry groups, in addition to knowing the physical particles that transformed under each group. Despite the great achievements of the Standard Model, it is known to fail for accommodating Dark Matter and its candidates. **In the next chapter**, we will talk about a very important physical model that is considered the simplest physical models extending from the Standard Model, as it was able to explain many important physical phenomena that the standard model could not provide an explanation for it, for example the explanation of the phenomenon of the presence of Dark Matter and its particles. This model is called the **Two Higgs Doublet Model (2HDMs)**, it was able to explain the existence of two doublets from the Higgs Field, unlike the Standard Model, which includes only one doublet from the Higgs Field. In the same chapter, we will also talk about a very special case of **(2HDMs)**, which is called the **Inert Doublet Model (IDM)**.

Chapter 4

Two Higgs Doublet Models (2HDMs)

4.1 Potential Of The Two Higgs Doublet

The presence of one doublet of the Higgs Field in the Standard Model is a good thing, but at the same time it is not enough to reveal other scientific facts. Therefore, the need for the presence of two doublet of the Higgs Field and also the presence of three doublet of the Higgs Field is a very important thing. The model (2HDMs) is considered one of the simplest extended models for the Standard Model, as it contains two Higgs doublet (Φ_1, Φ_2). This would help in the detection of very important physical phenomena in the near future, including the detection of the existence of Dark Matter and the interpretation of its particles. According to the Standard Model, the single doublet of Higgs does not have an electric charge **(uncharged)** but in the model (2HDMs), it was found that it contains five Higgs particles, including two Charged Higgs particles, they will be identified later in this chapter [49]. Regarding the model (2HDMs), there are many working groups, especially those are working in the particle physics **(High Energy Physics)** had done many scientific research on the possibility of finding a Charged Higgs and the researches for that were associated with the possibility of the existence of two doublet of the Higgs Field. In other words, these scientific research and physical work groups presented very impressive research on the possibility of detecting the charge of the Higgs that the Standard Model could not explain it [50]. These groups began since the beginning of the twentieth century in the search and detection the charge of Higgs, they made many primitive scientific experiments were carried out in the **Large Hadron Collider (LHC)** which gave an important explanations about the possibility of an electric charge of Higgs [51], these groups are four namely as **(ALEPH),(DELPHI),(L3),(OPAL)**. The scientific researches of them were able to explain that there are a close links between the charge of Higgs and the possibility of the existence of two doublet of Higgs Field [52], where they focused all their research and scientific reports on the subject of discovering a charge of Higgs because of its great importance in particle physics, one of the most important results of these researches that the energy mass of the Charged Higgs ranges between **40 - 100 Gev/$\mathbf{c^2}$** [53,54]. The several scenarios related to the subject of the charge of Higgs had been assumed, as these fall under the most important topic which is (2HDMs). Concerning the Two Doublet Model of the Higgs Field, it includes five physical particles of the Higgs that are as follows [55,56]:

- The CP - Even h.

- The CP - Even H.

- The CP - Odd A.
- The Charged Higgs Boson with Symbol (H^+).
- The Charged Higgs Boson with Symbol (H^-).

As for the acronym (**CP**), it is related to the **Spatial Charge Parity Symmetry**. Its an important symmetry that must be conserved in the majority of physical systems especially in Particle Physics in addition to not violating it. With regard to the previous five types, the two charged Higgs ($\boldsymbol{H^+, H^-}$) cannot represent dark matter particles because they are charged particles, contrary to what is known about dark matter as neutral matter that does not interact, while the three types of Higgs particles **(Even h , Even H , Odd A)** can represent dark matter particles [57, 58]. The (2HDMs) led also to the existence of many possible scenarios that would provide important explanations for the existence of important new physical particles that help in the development of Particle Physics in particular and Physics in general, among these scenarios, the correlation of the Charged Higgs with the Fermion particles carrying a spin of either $(+\frac{\mathbf{1}}{\mathbf{2}})$ or $(-\frac{\mathbf{1}}{\mathbf{2}})$. For the (2HDMs), the mathematical formula for its scalar potential is given as follows [59–62]:

$$\begin{aligned} V(\Phi_1,\Phi_2) = & -\frac{1}{2}\left(m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left(m_{12}^2\Phi_1^\dagger\Phi_2 + H.c.\right)\right) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 \\ & + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 \\ & + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + H.c. \end{aligned} \tag{4.1}$$

- $(\boldsymbol{\Phi_1}, \boldsymbol{\Phi_2})$: Two Higgs Field Doublets. $(\boldsymbol{\Phi_1^\dagger})$: Hermitian Conjugate Of Higgs Doublet Φ_1.
- $(\boldsymbol{\Phi_2^\dagger})$: Hermitian Conjugate Of Higgs Doublet Φ_2. $(\mathbf{m_{11}^2}, \mathbf{m_{22}^2})$: These Coefficients are Real.
- $(\lambda_{1,2,3,4})$: These Coefficients are Real. $(\mathbf{m_{12}^2})$: This Coefficient is Complex.
- $(\lambda_{5,6,7})$: These Coefficients are Complex. **(H.c)**: Hermitian Conjugate Of Others Terms.

Just as the previous mathematical formula for the related scalar potential that has been written (4.1), in addition to all its symbols being clarified, the matrix form of Two Higgs Doublets must also be known. This formula will be written as follows [63, 64]:

$$\Phi_i = \begin{pmatrix} \varphi_i^+ \\ \frac{1}{\sqrt{2}}(\rho_i + \eta_i + i\chi_i) \end{pmatrix} \quad , \quad i = 1, 2 \tag{4.2}$$

- Φ_i , φ_i^+: Higgs Field Doublet and Complex Field respectively.
- ρ_i: It represents Vacuum Expectation Value.
- η_i , χ_i: They represent Real Field.

- For the previous matrix formula (4.2), both Higgs Doublet (Φ_1) and (Φ_2) will be written as follows:

$$\Phi_1 = \begin{pmatrix} \varphi_1^+ \\ \frac{1}{\sqrt{2}}(\rho_1 + \eta_1 + i\chi_1) \end{pmatrix} \tag{4.3}$$

$$\Phi_2 = \begin{pmatrix} \varphi_2^+ \\ \frac{1}{\sqrt{2}}(\rho_2 + \eta_2 + i\chi_2) \end{pmatrix} \tag{4.4}$$

- For the previous two formulas, it can be also written the dagger matrix form ($\Phi_1^\dagger$) and ($\Phi_2^\dagger$). These new two formulas will be written as follows:

$$\Phi_1^\dagger = \begin{pmatrix} \varphi_1^- & \frac{1}{\sqrt{2}}(\rho_1 + \eta_1 - i\chi_1) \end{pmatrix} \tag{4.5}$$

$$\Phi_2^\dagger = \begin{pmatrix} \varphi_2^- & \frac{1}{\sqrt{2}}(\rho_2 + \eta_2 - i\chi_2) \end{pmatrix} \tag{4.6}$$

- Regarding the **Vacuum Expectation Value (VEV)** related to the Standard Model (ν). This value must satisfy the following physical equation [65, 66]:

$$\nu = \sqrt{\rho_1^2 + \rho_2^2} \approx 246\ GeV \tag{4.7}$$

- The angle between the Higgs doublet ($\boldsymbol{\Phi_1}, \boldsymbol{\Phi_2}$) will be found through the vital following relationship [65, 66]:

$$\tan(\beta) = \frac{\rho_2}{\rho_1} \tag{4.8}$$

4.2 Total Lagrangian Density Equation For (2HDMs)

The **T**otal **L**agrangian **E**quation especially in particle physics plays an important role in revealing how strong and weak interactions occur between particles such as Fermions, Bosons and other particles of the Standard Model. This require writing both the **K**inetic **E**nergy **E**quation and the **P**otential **E**nergy **E**quation for these particles and then doing special mathematical operations such as derivation and integration in order to obtain the correct interpretations of the models that it is been studied [68–71]. The Lagrangian Equation for (2HDMs) will be in the following form:

$$\ell = \ell_{gf}^{SM} + \ell_{Yukawa} + \ell_{Higgs} \tag{4.9}$$

- ℓ: Total Lagrangian Equation.
- ℓ_{gf}^{SM}: Standard Model Interaction of Fermions and Gauge Bosons (Force Carriers).
- ℓ_Y: Yukawa Interaction of Fermions with Higgs Field.
- ℓ_{Higgs}: Higgs Field Lagrangian.

The last term of equation (4.9) is considered the most important term that explains the Higgs Interactions (ℓ_{Higgs}). The Lagrangian Equation for the Higgs Field consists of two parts, one of which is related to Kinetic Term (T_H), while the other is related to Potential Term (V_H). They will be explained in detail as follows [72–74]:

$$\ell_{Higgs} = T_H - V_H \tag{4.10}$$

- ℓ_{Higgs}: The Higgs Field Lagrangian.
- T_H: The Kinetic Term of the Higgs Field.
- V_H: The Potential Term of the Higgs Field.

$$\begin{aligned} T_H = & (D_{1\mu}\Phi_1)^\dagger(D_1^\mu\Phi_1) + (D_{2\mu}\Phi_2)^\dagger(D_2^\mu\Phi_2) \\ & + \chi\ (D_{1\mu}\Phi_1)^\dagger(D_2^\mu\Phi_2) + \chi^*\ (D_{2\mu}\Phi_2)^\dagger(D_1^\mu\Phi_1) \end{aligned} \tag{4.11}$$

$$D_{1\mu} = \partial_\mu\ +\ i\frac{g_1}{2}\sigma_i W_\mu^i\ +\ i\frac{\acute{g}_1}{2}B_\mu \tag{4.12}$$

$$D_{2\mu} = \partial_\mu\ +\ i\frac{g_2}{2}\sigma_i W_\mu^i\ +\ i\frac{\acute{g}_2}{2}B_\mu \tag{4.13}$$

The Potential Term (V_H) of the Higgs Field will be interpreted as follows:

$$V_H =\ V_1\ +\ V_2\ +\ V_{int} \tag{4.14}$$

- V_H: The Potential Term of the Higgs Field.
- V_1: The Potential of Higgs Field (Φ_1).
- V_2: The Potential of Higgs Field (Φ_2).
- V_{int}: The Interaction Potential of Fields (Φ_1) and (Φ_2).

Each of the terms in equation (4.14) has its very own equation that plays an important role in describing the **Interactions between Two Higgs Doublet**. The last term of equation (4.14) is the most important term ($\mathbf{V_{int}}$). It shows the extent to which the first doublet of Higgs Field ($\mathbf{\Phi_1}$) interacts with the second doublet of Higgs Field ($\mathbf{\Phi_2}$). The equation of this term ($\mathbf{V_{int}}$) will be clarified as follows [75–77]:

$$\begin{aligned} V_{int} = & \; m_{12}^2(\Phi_1^\dagger\Phi_2) \;+\; (m_{12}^2)^*(\Phi_2^\dagger\Phi_1) \;+\; \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\ & + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \;+\; \frac{1}{2}\,[\,\lambda_5(\Phi_1^\dagger\Phi_2)^2 \;+\; \lambda_5^*(\Phi_2^\dagger\Phi_1)^2\,] \\ & + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) \;+\; \lambda_6^*(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) \\ & + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) \;+\; \lambda_7^*(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \end{aligned} \tag{4.15}$$

4.3 Inert Doublet Model (IDM)

The Inert Doublet Model is considered one of the important models in the history of particle physics in particular and in general in the history of physics. Its importance in providing very important explanations about physical phenomena that the Standard Model could not put an explanation for these phenomena such as the explanation of Dark Matter and its particles [78]. The Standard Model contain only one doublet of the Higgs Boson while the (IDM) model was able to explain the existence of two doublets of the Higgs Boson, this led to the possibility of detecting the presence of physical particles other than those in the Standard Model. The (IDM) model was introduced in the seventies (1970s) and the concept of it developed to be able to generate light neutron particles within the Tera Electron Volt **(TeV)** range in order to achieve **Electroweak Symmetry Breaking** [79]. The (IDM) is considered a special case of the Two Higgs Doublet Model (2HDMs) and it's considered as a narrower version of the larger version (2HDMs) in the process of the interpretation of Dark Matter Candidates, in addition to helping generate neutrinos of different masses through the work of many mechanisms necessary for this. During this model, the topic of Dark Matter Candidates masses was addressed, where these masses have two regions, one of them is a Light Mass Region of about $\mathbf{5\ GeV/c^2}$ while the other region is an intermediate mass region of about $\mathbf{100\ GeV/c^2}$ [80]. The mass of Heavy Bosons can be up to $\mathbf{500\ GeV/c^2}$ and maybe more. All mathematical physical calculations in this model are based on how to calculate the **Relic Density** of Dark Matter, where these calculations require a computer program called the **Micro-Omega** software and also require the presence of many coupling constants (λ_L) [81].

The (IDM) consist of Two Higgs Doublet each of them has a special symbol, the first doublet has its symbol $(\mathbf{\Phi_S})$ while the second doublet has its symbol $(\mathbf{\Phi_D})$. Each doublet has its own distinctive properties where these properties play an important role in helping to detect Dark Matter Candidates [82]. Through the mathematical matrix formula for each of the doublets of (IDM), it can be concluded that one of them has a relationship to the vacuum expectation value of the Standard Model [83]. The (IDM) Doublets will be as the following:

- The First Doublet $(\mathbf{\Phi_S})$ **(Active Doublet)**: It has a certain vacuum expectation value that helps break the Electroweak Symmetry in the Standard Model [84].

- The Second Doublet $(\mathbf{\Phi_D})$ **(Inert Doublet)**: It has nothing to do with the value of the expectation of vacuum and is **not** associated with Fermions in the Standard Model [84].

The **Real Scalar Potential** of the Inert Doublet Model model will be clarified as follows:

$$\begin{aligned} V(\Phi_S, \Phi_D) = & -\frac{1}{2}\left(m_{11}^2(\Phi_S^\dagger \Phi_S) \ + \ m_{22}^2(\Phi_D^\dagger \Phi_D)\right) + \ \frac{\lambda_1}{2}(\Phi_S^\dagger \Phi_S)^2 \ + \ \frac{\lambda_2}{2}(\Phi_D^\dagger \Phi_D)^2 \\ & + \ \lambda_3(\Phi_S^\dagger \Phi_S)(\Phi_D^\dagger \Phi_D) \ + \ \lambda_4(\Phi_S^\dagger \Phi_D)(\Phi_D^\dagger \Phi_S) + \ \frac{\lambda_5}{2}\left((\Phi_S^\dagger \Phi_D)^2 \ + \ (\Phi_D^\dagger \Phi_S)\right) \end{aligned} \tag{4.16}$$

- $(\mathbf{\Phi_S})$: Active Doublet Of IDM. $(\mathbf{\Phi_D})$: Inert Doublet Of IDM.
- $(\mathbf{\Phi_S^\dagger})$: Hermitian Conjugate Of Active Doublet Φ_S.
- $(\mathbf{\Phi_D^\dagger})$: Hermitian Conjugate Of Inert Doublet Φ_D.
- $(\lambda_{1,2,3,4,5})$: These Coupling Constants are Real . $(\mathbf{m_{11}^2} \ , \ \mathbf{m_{22}^2})$: These Constants are Real.

The Matrix Mathematical Formula For (IDM) Doublets will be explained as follows [85]:

$$\Phi_S = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}} \ (\nu + h + i\xi) \end{pmatrix} \quad , \quad \Phi_D = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} \ (H + iA) \end{pmatrix} \tag{4.17}$$

$$\Phi_S^\dagger = \begin{pmatrix} \phi^- & \frac{1}{\sqrt{2}} \ (\nu + h - i\xi) \end{pmatrix} \quad , \quad \Phi_D^\dagger = \begin{pmatrix} H^- & \frac{1}{\sqrt{2}} \ (H - iA) \end{pmatrix} \tag{4.18}$$

- $(\phi^+ \ , \ \phi^-)$: Hermitian Conjugate Fields in the Doublets (Φ_S) and $(\Phi_S^\dagger)$ respectively.
- (ν): Standard Model's Vacuum Expectation Value, which is **(246)** Gev.
- $(\mathbf{H}, \mathbf{A}, \mathbf{H}^+, \mathbf{H}^-)$: Four Scalar Fields (Inert or Dark).
- (ξ): It represents Real Field. $(\mathbf{h})$: It represents Higgs Boson.

As for the Second Doublet $(\mathbf{\Phi_D})$, it was previously explained that it contains **Four Dark Scalar Fields** $(\mathbf{H} \ , \ \mathbf{A} \ , \ \mathbf{H}^+, \mathbf{H}^-)$. These fields have special equations related to the mass of their particles in each field of them, in addition to that these masses have a direct relationship to the **Coupling Constants** (λ). Therefore, the mass of each field will be explained as follows [86]:

$$Dark\ Scalar\ Fields\ (H^{\pm})\ Masses \ = \ m_{H^{\pm}}^2 \ = \ \frac{1}{2}\left(\lambda_3 \nu^2 - m_{22}^2\right) \tag{4.19}$$

$$Higgs\ Boson\ Mass \ = \ m_h^2 \ = \ m_{11}^2 \ = \ \lambda_1 \ \nu^2 \tag{4.20}$$

$$m_{22}^2 \ = \ \lambda_{345} \ \nu^2 \ - \ 2 \ m_H^2 \tag{4.21}$$

While the rest of the other Dark Scalar Field Masses (A) , (H) will be as follows:

$$\begin{aligned} \textit{Dark Scalar Field } (A) \textit{ Mass } = m_A^2 &= m_{H^\pm}^2 + \frac{1}{2}(\lambda_4 - \lambda_5)\nu^2 \\ &= \frac{1}{2}(\lambda_3\nu^2 - m_{22}^2) + \frac{1}{2}(\lambda_4 - \lambda_5)\nu^2 \\ &= \frac{1}{2}(\lambda_3\nu^2 + \lambda_4\nu^2 - \lambda_5\nu^2 - m_{22}^2) \\ &= \frac{1}{2}\Big((\lambda_3 + \lambda_4 - \lambda_5)\nu^2 - m_{22}^2\Big) \\ &= \frac{1}{2}(\bar{\lambda}_{345}\nu^2 - m_{22}^2) \end{aligned} \tag{4.22}$$

$$\begin{aligned} \textit{Dark Scalar Field } (H) \textit{ Mass } = m_H^2 &= m_{H^\pm}^2 + \frac{1}{2}(\lambda_4 + \lambda_5)\nu^2 \\ &= \frac{1}{2}(\lambda_3\nu^2 - m_{22}^2) + \frac{1}{2}(\lambda_4 + \lambda_5)\nu^2 \\ &= \frac{1}{2}(\lambda_3\nu^2 + \lambda_4\nu^2 + \lambda_5\nu^2 - m_{22}^2) \\ &= \frac{1}{2}\Big((\lambda_3 + \lambda_4 + \lambda_5)\nu^2 - m_{22}^2\Big) \\ &= \frac{1}{2}(\lambda_{345}\nu^2 - m_{22}^2) \end{aligned} \tag{4.23}$$

$$\bar{\lambda}_{345} = \lambda_3 + \lambda_4 - \lambda_5 \tag{4.24}$$

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5 \tag{4.25}$$

In Eq (4.17), it is clear that the doublet $(\mathbf{\Phi_D})$ of (IDM) has no relation to the Vacuum Expectation Value of the Standard Model, in addition to its particles do not interact with the fermions of the Standard Model. As a result, the particles of $(\mathbf{\Phi_D})$ may not decay and may be Dark Matter Candidates [87]. Regarding the vital special coupling constants (λ_L) in this model, it is clear that they are closely related to the mass of the Higgs Boson and the masses of the four dark scalar fields particles, it can be concluded the relationship of these (λ_L) with the special particles present in the doublet $(\mathbf{\Phi_D})$ [88]. The importance of each of them will be briefly explained as follows:

- λ_1: This Coupling Constant is only related to the mass of the Higgs Boson in the Standard Model [89–91].

- λ_2: This Coupling Constant describes the all Self Coupling of the Four Dark Scalar Field Particles that exist in the doublet (Φ_D) [89–91].

- λ_3: This Coupling Constant describes the interactions between the Higgs Boson (h) in the doublet (Φ_S) and the Scalar Charged Dark Field Particles $(H^\pm)$ in the doublet (Φ_D) [89–91].

- λ_{345}: This Coupling Constant describes the interactions that occur between the Higgs Boson of the doublet (Φ_S) with the Dark Scalar Field Particles (H) of the doublet (Φ_D) [89–91].

- $\bar{\lambda}_{345}$: This Coupling Constant describes the interactions that occur between the Higgs Boson of the doublet (Φ_S) with the Dark Scalar Field Particles (A) of the doublet (Φ_D) [89–91].

4.4 Summary

Finally, the end of this chapter has been reached, where all the necessary scientific information and physical equations have been reviewed about the simplest physical models considered as an extension of the Standard Model called (2HDMs), in addition to that the special case of the model (2HDMs) which is called the Inert Double Model (IDM) has been also clarified. The subject of the physical Lagrangian Density Equation, which is specific to the (2HDMs) model, was also addressed. The Lagrangian equation for the Higgs Field was also discussed, where it's both terms kinematic equation and potential equation were presented and explained, these two equations represent the basis of the Lagrangian Equation. Through them, the motion of Higgs particles will be identified, in addition to new Higgs particles masses will may be detect from their important potential equation. **In the next chapter**, we will talk about another physical model that also extends from the Standard Model, as it can be described as a more complex and comprehensive model than the previous model (2HDMs), this model is called the **Three Higgs Doublet Models (3HDMs)**, it contain three doublets of Higgs Field (Φ_1, Φ_2, Φ_3) and its scalar potential will be more complicated. The Scalar Potential Equation for (3HDMs) will be also explained in terms of both old basis (Φ_1, Φ_2, Φ_3) and new basis (h_1, h_2, h_s). After that, the S_3 symmetry will be presented in this model to show how the transformation between old basis and new basis will be occurred. The (3HDMs) consist of two vaccum configurations, which named as **Real Vacuum Configurations** and **Complex Vacuum Configurations**, these configurations will be clarified with thier own tables. Finally, we will review the Large Hadron Collider **(LHC)** and then clarify its parts, especially the most important part called ATLAS Detector, which is considered primarily responsible for the majority of physical experiments and scientific research related to particle physics. A scientific research conducted on the ATLAS Detector will also be discussed, which gives a large range of Dark Matter Masses.

Chapter 5

Three Higgs Doublet Models (3HDMs)

5.1 Motivations For Introducing (3HDMs)

It is known that the Standard Model contains one doublet of the Higgs Boson, while the model (2HDMs) contains two doublets of the Higgs Boson, which consist of five Higgs particles (Even h , Even H , Odd A , Charged Higgs Boson (H^+) , Charged Higgs Boson (H^-)). All of that led to the presence of many important motives for the existence of a physical model that is broader than both the Standard Model and (2HDMs) which is Three Higgs Doublet Models (3HDMs). The main motivation for physicists building the model (3HDMs) is the possibility of obtaining the largest number of Higgs boson particles as possible, which can accommodate Dark Matter particles. Another important motive is that the Standard Model contains three quark families (up and down), (charm and strange), (top and bottom), it also contains three lepton families (electron and electron neutrino), (muon and muon neutrino), (tau and tau neutrino), so why are there not also three families or three doublets of the Higgs Field. Table (5.1) Shows a comparison between the Standard Model, Two Higgs Doublet Models (2HDMs) and Three Higgs Doublet Models (3HDMs).

	Standard Model	**2HDMs**	**3HDMs**
Higgs Doublets	**One Doublet** (Φ)	(Φ_1, Φ_2)	(Φ_1, Φ_2, Φ_3)
Higgs Particles	**One Particle**	**5**	**Above 8**
Does it give Dark Matter Particles ?	**No**	**Yes**	**Yes**

Table 5.1: The Comparison between Standard Model, 2HDMs and 3HDMs.

5.2 Scalar Potential For (3HDMs)

This model is considered one of the most extensive and advanced physical models in particle physics. As it is considered one of the wonderful extensions of the Standard Model in explaining many physical phenomena such as Dark Matter and additional physical particles other than those in the Standard Model. This model differs from (IDM) and (2HDMs), it is more complex and expanded, as it includes three doublets of the Higgs Field, unlike each of the Standard Model, which includes one doublet of the Higgs Field, while the two models (IDM) and (2HDMs) contain two doublets of the Higgs Field. The basic idea in this model is the possibility of detecting the presence of additional physical particles from the Higgs Field particles in order to reach a reasonable physical explanation for many physical phenomena regarding both Dark Matter and Dark Energy in addition to other very important topics in particle physics. The doublets of the Higgs Field in this model are (Φ_1, Φ_2, Φ_3), as the mathematical matrix formula for these doublets will be explained as follows:

$$\Phi_i = \begin{pmatrix} \varphi_i^+ \\ \frac{1}{\sqrt{2}}(\rho_i + \eta_i + i\chi_i) \end{pmatrix} \tag{5.1}$$

- Each doublet of the three doublets of the Higgs field would be written as:

$$\Phi_1 = \begin{pmatrix} \varphi_1^+ \\ \frac{1}{\sqrt{2}}(\rho_1 + \eta_1 + i\chi_1) \end{pmatrix} \tag{5.2}$$

$$\Phi_2 = \begin{pmatrix} \varphi_2^+ \\ \frac{1}{\sqrt{2}}(\rho_2 + \eta_2 + i\chi_2) \end{pmatrix} \tag{5.3}$$

$$\Phi_3 = \begin{pmatrix} \varphi_3^+ \\ \frac{1}{\sqrt{2}}(\rho_3 + \eta_3 + i\chi_3) \end{pmatrix} \tag{5.4}$$

- For the previous three matrix formulas, it will be also written the hermitian conjugate matrix for these three Higgs doublets as symbols $(\Phi_1^\dagger)$, $(\Phi_2^\dagger)$ and $(\Phi_3^\dagger)$. These new three formulas will be written as follows:

$$\Phi_1^\dagger = \begin{pmatrix} \varphi_1^- & \frac{1}{\sqrt{2}}(\rho_1 + \eta_1 - i\chi_1) \end{pmatrix} \tag{5.5}$$

$$\Phi_2^\dagger = \begin{pmatrix} \varphi_2^- & \frac{1}{\sqrt{2}}(\rho_2 + \eta_2 - i\chi_2) \end{pmatrix} \tag{5.6}$$

$$\Phi_3^\dagger = \begin{pmatrix} \varphi_3^- & \frac{1}{\sqrt{2}}(\rho_3 + \eta_3 - i\chi_3) \end{pmatrix} \tag{5.7}$$

- With regard to the Vacuum Expectation Value **(VEV)** that relates to the standard model, this value is approx to a constant value of **(246 Gev)**. The correct equation relating this value to the Higgs field doublets will be written in the following form:

$$\nu = VEV = \sqrt{\rho_1^2 + \rho_2^2 + \rho_3^2} \approx 246\ GeV \tag{5.8}$$

For many particle physics topics to be studied and explained, it is easier to make transformations from an old basis to a new basis using special transformation matrices for that purpose. The goal of doing this transformation process is to write the physical equations very smoothly in order to obtain correct and accurate interpretations.

The **Total Scalar Potential** For (3HDMs) in basis (Φ_1, Φ_2, Φ_3) will be written as follows [92]:

$$\text{The Total Scalar Potential } (V) \;=\; V_2 \;+\; V_4 \tag{5.9}$$

$$\begin{aligned} V_2 \;&= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + m_{33}^2 \Phi_3^\dagger \Phi_3 \\ &\quad - \Big[m_{12}^2 (\Phi_1^\dagger \Phi_2) + m_{13}^2 (\Phi_1^\dagger \Phi_3) + m_{23}^2 (\Phi_2^\dagger \Phi_3) + h.c \Big] \end{aligned} \tag{5.10}$$

$$\begin{aligned} V_4 \;&= \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_3^\dagger \Phi_3)^2 + \lambda_4 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\ &\quad + \lambda_5 (\Phi_1^\dagger \Phi_1)(\Phi_3^\dagger \Phi_3) + \lambda_6 (\Phi_2^\dagger \Phi_2)(\Phi_3^\dagger \Phi_3) + \lambda_7 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ &\quad + \lambda_8 (\Phi_1^\dagger \Phi_3)(\Phi_3^\dagger \Phi_1) + \lambda_9 (\Phi_2^\dagger \Phi_3)(\Phi_3^\dagger \Phi_2) \end{aligned} \tag{5.11}$$

With regard to equation (5.9), it does not give a distinction between the three doublets of Higgs Field (Φ_1, Φ_2, Φ_3), in other words, it does not specify which of the doublets behaves as a Higgs Field Doublet of the Standard Model and which doublets are inert that can produce inert scalar particles that accommodate Dark Matter particles. As a result of this important issue, a transformation from the old bases (Φ_1, Φ_2, Φ_3) to the new bases (h_1, h_2, h_s) was carried out through a special transformation matrix under S_3 symmetry. The main goal of this transformation process is to be able to distinguish the doublets of the Higgs Field, especially those related to Dark Matter particles, in addition to reducing the number of coupling constants in order to write the scalar potential equation (5.9) in a good, uncomplicated way. In this model (3HDMs), the original Higgs Field Doublets with the old basis are (Φ_1, Φ_2, Φ_3), but after making the physical transformation under S_3 symmetry, these doublets with the new basis became (h_1, h_2, h_s). The transformation process between the old base and the new base takes place through a special transformation matrix. It will be written as follows [93]:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_s \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{pmatrix} \tag{5.12}$$

Through the physical transformation by equation (5.12), it is possible to find relationships between the old basis and the new basis by performing the process of multiplying the matrices and then obtaining the required physical equations that will be written in the following form:

$$h_1 \;=\; \frac{1}{\sqrt{2}}\,\Phi_1 - \frac{1}{\sqrt{2}}\,\Phi_2 \;=\; \frac{1}{\sqrt{2}}\,(\Phi_1 - \Phi_2) \tag{5.13}$$

$$h_2 \;=\; \frac{1}{\sqrt{6}}\,\Phi_1 + \frac{1}{\sqrt{6}}\,\Phi_2 - \frac{2}{\sqrt{6}}\,\Phi_3 \;=\; \frac{1}{\sqrt{6}}\,(\Phi_1 + \Phi_2 - 2\Phi_3) \tag{5.14}$$

$$h_s \;=\; \frac{1}{\sqrt{3}}\,\Phi_1 + \frac{1}{\sqrt{3}}\,\Phi_2 + \frac{1}{\sqrt{3}}\,\Phi_3 \;=\; \frac{1}{\sqrt{3}}\,(\Phi_1 + \Phi_2 + \Phi_3) \tag{5.15}$$

Previously, many important topics were discussed regarding the transformation from the old basis to the new basis of the three Higgs doublets. In the following, the important physical equation that forms the main part of this thesis will be explained. Its called The **Total Scalar Potential** For (3HDMs) in basis (h_1, h_2, h_s) with S_3 symmetry and it will be written as follows [94]:

$$\text{The Total Scalar Potential } (V) = V_2 \; + \; V_4 \tag{5.16}$$

$$V_2 \;=\; \mu_1^2\Big(h_1^\dagger h_1 + h_2^\dagger h_2\Big) + \mu_0^2 h_s^\dagger h_s \tag{5.17}$$

$$\begin{aligned} V_4 \;=& \lambda_1 \left(h_1^\dagger h_1 + h_2^\dagger h_2\right)^2 + \lambda_2 \left(h_1^\dagger h_2 - h_2^\dagger h_1\right)^2 + \lambda_3 \left[\left(h_1^\dagger h_1 - h_2^\dagger h_2\right)^2 \;+\; \left(h_1^\dagger h_2 + h_2^\dagger h_1\right)^2\right] \\ & + \lambda_4 \left[\left(h_s^\dagger h_1\right)\left(h_1^\dagger h_2 + h_2^\dagger h_1\right) \;+\; \left(h_s^\dagger h_2\right)\left(h_1^\dagger h_1 - h_2^\dagger h_2\right) \;+\; h.c\right] \\ & + \lambda_5 \left[\left(h_s^\dagger h_s\right)\left(h_1^\dagger h_1 + h_2^\dagger h_2\right)\right] \;+\; \lambda_6 \left[(h_1^\dagger h_s)\,(h_s^\dagger h_1) \;+\; (h_2^\dagger h_s)\,(h_s^\dagger h_2)\right] \\ & + \lambda_7 \left[(h_s^\dagger h_1)^2 \;+\; (h_s^\dagger h_2)^2 \;+\; h.c\right] \;+\; \lambda_8 \,(h_s^\dagger h_s)^2 \end{aligned} \tag{5.18}$$

This is the most general potential which has **10** vital independent parameters: Two Quadratic Coefficients μ_1^2 and μ_0^2 which dimensions are **Squared Mass** and Eight Quartic **Dimensionless** Coefficients $\lambda_1, ...\lambda_8$. Also, we notice this potential is symmetric under the interchange $h_1 \rightarrow -h_1$.

- We decompose the fields (h_1, h_2, h_s) as the following matrix formula:

$$h_1 = \begin{pmatrix} h_1^+ \\ \frac{1}{\sqrt{2}}(w_1 + \eta_1 + i\chi_1) \end{pmatrix} \tag{5.19}$$

$$h_2 = \begin{pmatrix} h_2^+ \\ \frac{1}{\sqrt{2}}(w_2 + \eta_2 + i\chi_2) \end{pmatrix} \tag{5.20}$$

$$h_s = \begin{pmatrix} h_s^+ \\ \frac{1}{\sqrt{2}}(w_s + \eta_s + i\chi_s) \end{pmatrix} \tag{5.21}$$

- For the previous three matrix formulas, it will be also written the hermitian conjugate matrix for these new three Higgs doublets as symbols $(h_1^\dagger)$, $(h_2^\dagger)$ and $(h_3^\dagger)$. These new three formulas will be written as follows:

$$h_1^\dagger = \begin{pmatrix} h_1^- & \frac{1}{\sqrt{2}}(w_1 + \eta_1 - i\chi_1) \end{pmatrix} \tag{5.22}$$

$$h_2^\dagger = \begin{pmatrix} h_2^- & \frac{1}{\sqrt{2}}(w_2 + \eta_2 - i\chi_2) \end{pmatrix} \tag{5.23}$$

$$h_s^\dagger = \begin{pmatrix} h_s^- & \frac{1}{\sqrt{2}}(w_s + \eta_3 - i\chi_3) \end{pmatrix} \tag{5.24}$$

- The Vacuum Expectation Values (VEVs) are related as follows:

$$w_1 = \frac{1}{\sqrt{2}}(\rho_1 - \rho_2) \tag{5.25}$$

$$w_2 = \frac{1}{\sqrt{6}}(\rho_1 + \rho_2 - 2\rho_3) \tag{5.26}$$

$$w_s = \frac{1}{\sqrt{3}}(\rho_1 + \rho_2 + \rho_3) \tag{5.27}$$

After the scalar potential equation for (3HDMs) has been written, both real vacuum configurations as well as complex vacuum configurations must be considered. Since this model requires the identification of previous vacuum configurations in order to reach physical explanations for Dark Matter and then detect its particles.

5.3 Real Vacuum Configurations For (3HDMs)

This type of vacuum configuration is a very important type, as it is only related to the real parts of the physical fields (w_1, w_2, w_s) and has nothing to do with the complex parts. In order to understand real vacuum configurations, the following mathematical derivatives must be made in order to obtain minimization relations [95, 96].

$$\frac{\partial V}{\partial w_1} \; , \; \frac{\partial V}{\partial w_2} \; , \; \frac{\partial V}{\partial w_s} \tag{5.28}$$

$$\begin{aligned} \frac{\partial V}{\partial w_1} \; &= \frac{1}{2}\mu_1^2 w_1^\star \; + \; \frac{1}{2}\lambda_1 w_1^\star \Big(\mid w_1 \mid^2 \; + \; \mid w_2 \mid^2 \Big) \; + \; \frac{1}{2}\lambda_2 \Big(w_1 w_2^{\star 2} - w_1^\star \mid w_2 \mid^2 \Big) \\ &+ \; \frac{1}{2}\lambda_3 \Big(w_1^\star \mid w_1 \mid^2 + w_1 w_2^{\star 2} \Big) \; + \; \frac{1}{2}\lambda_4 \Big(w_1 w_2^\star w_s^\star + w_1^\star w_2 w_s^\star + w_1^\star w_2^\star w_s \Big) \\ &+ \; \frac{1}{4}\Big(\lambda_5 + \lambda_6 \Big) w_1^\star \mid w_s \mid^2 \; + \frac{1}{2}\lambda_7 w_1 w_s^{\star 2} \; = \; 0 \end{aligned} \tag{5.29}$$

$$
\begin{aligned}
\frac{\partial V}{\partial w_2} &= \frac{1}{2}\mu_1^2 w_2^\star \ + \ \frac{1}{2}\lambda_1 w_2^\star\Big(\mid w_1 \mid^2 + \mid w_2 \mid^2 \Big) \ + \ \frac{1}{2}\lambda_2\Big(w_1^{\star 2} w_2 - \mid w_1 \mid^2 w_2\Big) \\
&+ \ \frac{1}{2}\lambda_3\Big(w_1^{\star 2} w_2 + w_2^\star \mid w_2 \mid^2 \Big) \ + \ \frac{1}{4}\lambda_4\Big(2w_s^\star(\mid w_1 \mid^2 - \mid w_2 \mid^2) + w_s(w_1^{\star 2} - w_2^{\star 2})\Big) \\
&+ \ \frac{1}{4}\Big(\lambda_5 + \lambda_6\Big) w_2^\star \mid w_s \mid^2 \ + \frac{1}{2}\lambda_7 w_2 w_s^{\star 2} \ = \ 0
\end{aligned}
\tag{5.30}
$$

$$
\begin{aligned}
\frac{\partial V}{\partial w_s} &= \frac{1}{2}\mu_0^2 w_s^\star \ + \ \frac{1}{4}\lambda_4\Big(2 \mid w_1 \mid^2 w_2^\star - w_2^\star \mid w_2 \mid^2 + w_1^{\star 2} w_2\Big) \\
&+ \ \frac{1}{4}\Big(\lambda_5 + \lambda_6\Big)\Big(\mid w_1 \mid^2 + \mid w_2 \mid^2 \Big) w_s^\star \\
&+ \ \frac{1}{2}\lambda_7\Big(w_1^{\star 2} + w_2^{\star 2}\Big) w_s \ + \ \frac{1}{2}\lambda_8 w_s^\star \mid w_s \mid^2 = \ 0
\end{aligned}
\tag{5.31}
$$

Just as it was reached on how to derive equations (5.29 , 5.30, 5.31) and then equate them to **zero** in order to obtain appropriate minimization relations **(constraints)**, we solve these three equations simultaneously to find the two quadratic coefficients μ_1^2 and μ_0^2 in terms of the quartic coefficients $\lambda_1, ...\lambda_8$. We find one solution for μ_0^2 which is:

$$
\mu_0^2 \ = \ \frac{1}{2w_s}[\lambda_4 w_2(w_2^2 - 3w_1^2) - w_s(\lambda_5 + \lambda_6 + 2\lambda_7)(w_1^2 + w_2^2) - 2\lambda_8 w_s^3] \tag{5.32}
$$

The two solutions for μ_1^2 will be in the following:

$$
\mu_1^2 \ = \ -\frac{1}{2}[2(\lambda_1 + \lambda_3)(w_1^2 + w_2^2) + 6\lambda_4 w_2 w_s + (\lambda_5 + \lambda_6 + 2\lambda_7)w_s^2] \tag{5.33}
$$

$$
\mu_1^2 \ = \ -\frac{1}{2}[2(\lambda_1 + \lambda_3)(w_1^2 + w_2^2) - 3\lambda_4(w_2^2 - w_1^2)(w_s/w_2) + (\lambda_5 + \lambda_6 + 2\lambda_7)w_s^2] \tag{5.34}
$$

The two solutions for μ_1^2 in Eqs (5.33 , 5.34) are **not** consistent. They can be consistent if:

$$
-3\lambda_4(w_2^2 - w_1^2)(w_s/w_2) = 6\lambda_4 w_2 w_s \tag{5.35}
$$

which can be satisfied in the following cases: $\lambda_4 = 0$ or $w_s = 0$ or $w_1^2 = 3w_2^2$. There is something very important will be addressed, which is the cases of real vacuum configurations, where for each case there are certain constraints that represent minimization relations, through these constraints will lead to the desired results related to dark matter particles. Table (5.2) will describe all cases of the real vacuum configuration with all vaccum expectation values (w_1, w_2, w_s) as well as the constraints for each cases [97].

Table (5.2) will describe all cases of the **Real Vacuum Configuration (RVC)** as follows:

Vacuum	w_1, w_2, w_s	**Constraints**
R-0	$(0\ ,\ 0\ ,\ 0)$	None
R-I-1	$(0\ ,\ 0\ ,\ w_s)$	$\mu_0^2 = -\lambda_8 w_s^2$
R-I-2a	$(w\ ,\ 0\ ,\ 0)$	$\mu_1^2 = -(\lambda_1+\lambda_3)w_1^2$
R-I-2b	$(w\ ,\ \sqrt{3}w\ ,\ 0)$	$\mu_1^2 = -\frac{4}{3}(\lambda_1+\lambda_3)w_2^2$
R-I-2c	$(w\ ,\ -\sqrt{3}w\ ,\ 0)$	$\mu_1^2 = -\frac{4}{3}(\lambda_1+\lambda_3)w_2^2$
R-II-1a	$(0\ ,\ w\ ,\ w_s)$	$\mu_0^2 = \frac{1}{2}\lambda_4\frac{w_2^3}{w_s} - \frac{1}{2}(\lambda_5+\lambda_6+2\lambda_7)w_2^2 - \lambda_8 w_s^2$, $\mu_1^2 = -(\lambda_1+\lambda_3)w_2^2 + \frac{3}{2}\lambda_4 w_2 w_s - \frac{1}{2}(\lambda_5+\lambda_6+2\lambda_7)w_s^2$
R-II-1b	$(w\ ,\ -\frac{w}{\sqrt{3}}\ ,\ w_s)$	$\mu_0^2 = -4\lambda_4\frac{w_2^3}{w_s} - 2(\lambda_5+\lambda_6+2\lambda_7)w_2^2 - \lambda_8 w_s^2$, $\mu_1^2 = -4(\lambda_1+\lambda_3)w_2^2 + 3\lambda_4 w_2 w_s - \frac{1}{2}(\lambda_5+\lambda_6+2\lambda_7)w_s^2$
R-II-1c	$(w\ ,\ \frac{w}{\sqrt{3}}\ ,\ w_s)$	$\mu_0^2 = -4\lambda_4\frac{w_2^3}{w_s} - 2(\lambda_5+\lambda_6+2\lambda_7)w_2^2 - \lambda_8 w_s^2$, $\mu_1^2 = -4(\lambda_1+\lambda_3)w_2^2 + 3\lambda_4 w_2 w_s - \frac{1}{2}(\lambda_5+\lambda_6+2\lambda_7)w_s^2$
R-II-2	$(0\ ,\ w\ ,\ 0)$	$\mu_1^2 = -(\lambda_1+\lambda_3)w_2^2$, $\lambda_4 = 0$
R-II-3	$(w_1\ ,\ w_2\ ,\ 0)$	$\mu_1^2 = -(\lambda_1+\lambda_3)(w_1^2+w_2^2)$, $\lambda_4 = 0$
R-III	$(w_1\ ,\ w_2\ ,\ w_s)$	$\mu_0^2 = -\frac{1}{2}(\lambda_5+\lambda_6+2\lambda_7)(w_1^2+w_2^2) - \lambda_8 w_s^2$, $\mu_1^2 = -(\lambda_1+\lambda_3)(w_1^2+w_2^2) - \frac{1}{2}(\lambda_5+\lambda_6+2\lambda_7)w_s^2$ $\lambda_4 = 0$

Table 5.2: The Possible Real Vacuum Configurations For 3HDMs [97].

5.4 Complex Vacuum Configurations For (3HDMs)

The second type of vacuum configurations are **Complex Vacuum Configurations**, where these configurations are complex and more difficult than **Real Vacuum Configurations**. In this type the physical fields (w_1, w_2, w_s) will be transformed into another complex form that has a form of its own and this would make the new constraints on the minimization relations resulting from the derivation more difficult. Each of the new physical fields $(\hat{w}_1, \hat{w}_2, \hat{w}_s)$ as well as the derivation equations for these configurations will be explained and written in the following form [95, 96]:

$$(w_1, w_2, w_s) \;\longrightarrow\; (\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_s) \tag{5.36}$$

$$\frac{\partial V}{\partial \hat{w}_1}\ ,\ \frac{\partial V}{\partial \hat{w}_2}\ ,\ \frac{\partial V}{\partial \hat{w}_s}\ ,\ \frac{\partial V}{\partial \sigma_1}\ ,\ \frac{\partial V}{\partial \sigma_2}\ . \tag{5.37}$$

There are two important symbols in the two equations (5.36 , 5.37) that that will be as follows:

- $\hat{w}_i$: This Symbol represents the Absolute Value of w_i.
- σ_i: This Symbol represents the Transformation Phase.

All partial derivatives in equation (5.37) for Complex Vacuum Configuration will be as follows:

$$\begin{aligned} \frac{\partial V}{\partial \hat{w}_1} &= \mu_1^2 \hat{w}_1 \;+\; \lambda_1 \hat{w}_1 \Big(\hat{w}_1{}^2 + \hat{w}_2{}^2 \Big) \;-\; 2\lambda_2 \hat{w}_1 \hat{w}_2{}^2 s^2_{\sigma_1-\sigma_2} \;+\; \lambda_3 \hat{w}_1 \Big(\hat{w}_1{}^2 + \hat{w}_2{}^2 c_{2(\sigma_1-\sigma_2)} \Big) \\ &\quad +\; \lambda_4 \hat{w}_1 \hat{w}_2 \hat{w}_s \Big(c_{2\sigma_1-\sigma_2} + 2c_{\sigma_2} \Big) \;+\; \frac{1}{2}\Big(\lambda_5 + \lambda_6 \Big) \hat{w}_1 \hat{w}_s{}^2 \;+\; \lambda_7 \hat{w}_1 \hat{w}_s{}^2 c_{2\sigma_1} \;=\; 0 \end{aligned} \tag{5.38}$$

$$\begin{aligned} \frac{\partial V}{\partial \hat{w}_2} &= \mu_1^2 \hat{w}_2 \;+\; \lambda_1 \hat{w}_2 \Big(\hat{w}_1{}^2 + \hat{w}_2{}^2 \Big) \;-\; 2\lambda_2 \hat{w}_1{}^2 \hat{w}_2 s^2_{\sigma_1-\sigma_2} \;+\; \lambda_3 \hat{w}_2 \Big(\hat{w}_1{}^2 c_{2(\sigma_1-\sigma_2)} + \hat{w}_2{}^2 \Big) \\ &\quad +\; \frac{1}{2}\lambda_4 \hat{w}_s \Big(\hat{w}_1{}^2 c_{2\sigma_1-\sigma_2} + (2\hat{w}_1{}^2 - 3\hat{w}_2{}^2) c_{\sigma_2} \Big) \;+\; \frac{1}{2}\Big(\lambda_5 + \lambda_6 \Big) \hat{w}_2 \hat{w}_s{}^2 \\ &\quad +\; \lambda_7 \hat{w}_2 \hat{w}_s{}^2 c_{2\sigma_2} \;=\; 0 \end{aligned} \tag{5.39}$$

$$\begin{aligned} \frac{\partial V}{\partial \hat{w}_s} &= \mu_0^2 \hat{w}_s \;+\; \frac{1}{2}\lambda_4 \hat{w}_2 \Big(\hat{w}_1{}^2 c_{2\sigma_1-\sigma_2} + (2\hat{w}_1{}^2 - \hat{w}_2{}^2) c_{\sigma_2} \Big) \;+\; \frac{1}{2}\Big(\lambda_5 + \lambda_6 \Big) \Big(\hat{w}_1{}^2 + \hat{w}_2{}^2 \Big) \hat{w}_s \\ &\quad +\; \lambda_7 \hat{w}_s \Big(\hat{w}_1{}^2 c_{2\sigma_1} + \hat{w}_2{}^2 c_{2\sigma_2} \Big) \;+\; \lambda_8 \hat{w}_s{}^3 \;=\; 0 \end{aligned} \tag{5.40}$$

$$\frac{\partial V}{\partial \sigma_1} \;=\; -\Big(\lambda_2 + \lambda_3 \Big) \hat{w}_1{}^2 \hat{w}_2{}^2 s_{2(\sigma_1-\sigma_2)} \;-\; \lambda_4 \hat{w}_1{}^2 \hat{w}_2 \hat{w}_s s_{(2\sigma_1-\sigma_2)} \;-\; \lambda_7 \hat{w}_1{}^2 \hat{w}_s{}^2 s_{\sigma_1} \;=\; 0 \tag{5.41}$$

$$\begin{aligned} \frac{\partial V}{\partial \sigma_2} &= \Big(\lambda_2 + \lambda_3 \Big) \hat{w}_1{}^2 \hat{w}_2{}^2 s_{2(\sigma_1-\sigma_2)} \;+\; \frac{1}{2}\lambda_4 \hat{w}_2 \hat{w}_s \Big(\hat{w}_1{}^2 s_{2\sigma_1-\sigma_2} - (2\hat{w}_1{}^2 - \hat{w}_2{}^2) s_{\sigma_2} \Big) \\ &\quad -\; \lambda_7 \hat{w}_2{}^2 \hat{w}_s{}^2 s_{2\sigma_2} \;=\; 0 \end{aligned} \tag{5.42}$$

Just as how to obtain the derivation equations (5.38 , 5.39 , 5.40 , 5.41 , 5.42) for Complex Vacuum Configurations (CVC), the cases of these configurations will also be identified, in addition to the constraints of each case. Tables (5.3 , 5.4 , 5.5) will explain all possible cases of **Complex Vacuum Configurations (CVC)** as follows [97]:

Vacuum	w_1, w_2, w_s	**Constraints**
C-I-a	$(\hat{w}_1\ ,\ \pm i\hat{w}_1\ ,\ 0)$	$\mu_1^2 \ =\ -2(\lambda_1-\lambda_2){\hat{w}_1}^2$
C-III-a	$(0\ ,\ \hat{w}_2 e^{i\sigma_2}\ ,\ \hat{w}_s)$	$\mu_0^2 = -\frac{1}{2}(\lambda_5+\lambda_6-2\lambda_7){\hat{w}_2}^2 - \lambda_8{\hat{w}_s}^2\ ,$ $\mu_1^2 \ =\ -(\lambda_1+\lambda_3){\hat{w}_2}^2 - \frac{1}{2}(\lambda_5+\lambda_6-2\lambda_7-8c_{\sigma_2}^2\lambda_7){\hat{w}_s}^2\ ,$ $\lambda_4 \ =\ \frac{4c_{\sigma_2}\hat{w}_s}{\hat{w}_2}\lambda_7$
C-III-b	$(\pm i\hat{w}_1\ ,\ 0\ ,\ \hat{w}_s)$	$\mu_0^2 = -\frac{1}{2}(\lambda_5+\lambda_6-2\lambda_7){\hat{w}_1}^2 - \lambda_8{\hat{w}_s}^2\ ,$ $\mu_1^2 \ =\ -(\lambda_1+\lambda_3){\hat{w}_1}^2 - \frac{1}{2}(\lambda_5+\lambda_6-2\lambda_7){\hat{w}_s}^2\ ,$ $\lambda_4 \ =\ 0$
C-III-c	$(\hat{w}_1 e^{i\sigma_1}\ ,\ \hat{w}_2 e^{i\sigma_2}\ ,\ 0)$	$\mu_1^2 \ =\ -(\lambda_1+\lambda_3)({\hat{w}_1}^2+{\hat{w}_1}^2)$ $\lambda_2+\lambda_3 \ =\ 0\ ,$ $\lambda_4 \ =\ 0$
C-III-d	$(\pm i\hat{w}_1\ ,\ \hat{w}_2\ ,\ \hat{w}_s)$	$\mu_0^2 \ =\ (\lambda_2+\lambda_3)\frac{({\hat{w}_1}^2-{\hat{w}_2}^2)^2}{{\hat{w}_s}^2} - \frac{({\hat{w}_1}^2-{\hat{w}_2}^2)({\hat{w}_1}^2-3{\hat{w}_2}^2)}{4\hat{w}_2\hat{w}_s}\lambda_4$ $-\frac{1}{2}(\lambda_5+\lambda_6)({\hat{w}_1}^2+{\hat{w}_2}^2) - \lambda_8{\hat{w}_s}^2\ ,$ $\mu_1^2 \ =\ -(\lambda_1-\lambda_2)({\hat{w}_1}^2+{\hat{w}_2}^2) - \frac{\hat{w}_s({\hat{w}_1}^2-{\hat{w}_2}^2)}{4\hat{w}_2}\lambda_4,$ $-\frac{1}{2}(\lambda_5+\lambda_6){\hat{w}_s}^2\ ,$ $\lambda_7 \ =\ \frac{({\hat{w}_1}^2-{\hat{w}_2}^2)}{{\hat{w}_s}^2}(\lambda_2+\lambda_3) - \frac{({\hat{w}_1}^2-5{\hat{w}_2}^2)}{4\hat{w}_2\hat{w}_s}\lambda_4$
C-III-e	$(\pm i\hat{w}_1\ ,\ -\hat{w}_2\ ,\ \hat{w}_s)$	$\mu_0^2 \ =\ (\lambda_2+\lambda_3)\frac{({\hat{w}_1}^2-{\hat{w}_2}^2)^2}{{\hat{w}_s}^2} + \frac{({\hat{w}_1}^2-{\hat{w}_2}^2)({\hat{w}_1}^2-3{\hat{w}_2}^2)}{4\hat{w}_2\hat{w}_s}\lambda_4$ $-\frac{1}{2}(\lambda_5+\lambda_6)({\hat{w}_1}^2+{\hat{w}_2}^2) - \lambda_8{\hat{w}_s}^2\ ,$ $\mu_1^2 \ =\ -(\lambda_1-\lambda_2)({\hat{w}_1}^2+{\hat{w}_2}^2) + \frac{\hat{w}_s({\hat{w}_1}^2-{\hat{w}_2}^2)}{4\hat{w}_2}\lambda_4,$ $-\frac{1}{2}(\lambda_5+\lambda_6){\hat{w}_s}^2\ ,$ $\lambda_7 \ =\ \frac{({\hat{w}_1}^2-{\hat{w}_2}^2)}{{\hat{w}_s}^2}(\lambda_2+\lambda_3) + \frac{({\hat{w}_1}^2-5{\hat{w}_2}^2)}{4\hat{w}_2\hat{w}_s}\lambda_4$
C-III-f	$(\pm i\hat{w}_1\ ,\ i\hat{w}_2\ ,\ \hat{w}_s)$	$\mu_0^2 = -\frac{1}{2}(\lambda_5+\lambda_6-2\lambda_7)({\hat{w}_1}^2+{\hat{w}_2}^2) - \lambda_8{\hat{w}_s}^2\ ,$ $\mu_1^2 \ =\ -(\lambda_1+\lambda_3)(w_1^2+w_2^2) - \frac{1}{2}(\lambda_5+\lambda_6-2\lambda_7){\hat{w}_s}^2\ ,$ $\lambda_4 \ =\ 0$
C-III-g	$(\pm i\hat{w}_1\ ,\ -i\hat{w}_2\ ,\ \hat{w}_s)$	$\mu_0^2 = -\frac{1}{2}(\lambda_5+\lambda_6-2\lambda_7)({\hat{w}_1}^2+{\hat{w}_2}^2) - \lambda_8{\hat{w}_s}^2\ ,$ $\mu_1^2 \ =\ -(\lambda_1+\lambda_3)(w_1^2+w_2^2) - \frac{1}{2}(\lambda_5+\lambda_6-2\lambda_7){\hat{w}_s}^2\ ,$ $\lambda_4 \ =\ 0$

Table 5.3: table
The First Conditions Of Possible Complex Vacuum Configuration For 3HDMs [97].

Tabel (5.4) will explain the second conditions of **(CVC)** and it will be as the following:

Vacuum	w_1, w_2, w_s	**Constraints**
C-III-h	$(\sqrt{3}\hat{w}_2 e^{i\sigma_2}\ ,\ \pm\hat{w}_2 e^{i\sigma_2}\ ,\ \hat{w}_s)$	$\mu_0^2 = -2(\lambda_5+\lambda_6-2\lambda_7)\hat{w}_2{}^2 - \lambda_8\hat{w}_s{}^2$, $\mu_1^2 = -4(\lambda_1+\lambda_3)\hat{w}_2{}^2 - \frac{1}{2}(\lambda_5+\lambda_6-2\lambda_7-8c_{\sigma_2}^2\lambda_7)\hat{w}_s{}^2$, $\lambda_4 = \mp\frac{2c_{\sigma_2}\hat{w}_s}{\hat{w}_2}\lambda_7$
C-III-i	$(\sqrt{3}\hat{w}_2 e^{i\sigma_2}\ ,\ \pm\hat{w}_2 e^{i\sigma_2}\ ,\ \hat{w}_s)$	$\mu_0^2 = \frac{16(1-3t_{\sigma_1}^2)^2}{(1+9t_{\sigma_1}^2)^2}(\lambda_2+\lambda_3)\frac{\hat{w}_2{}^4}{\hat{w}_s{}^2} \pm \frac{6(1-t_{\sigma_1}^2)(1-3t_{\sigma_1}^2)}{(1+9t_{\sigma_1}^2)^{\frac{3}{2}}}\lambda_4\frac{\hat{w}_2{}^3}{\hat{w}_s}$ $-\frac{2(1+3t_{\sigma_1}^2)}{(1+9t_{\sigma_1}^2)}(\lambda_5+\lambda_6)\hat{w}_2{}^2 - \lambda_8\hat{w}_s{}^2$, $\mu_1^2 = -\frac{4(1+3t_{\sigma_1}^2)}{(1+9t_{\sigma_1}^2)}(\lambda_1-\lambda_2)\hat{w}_2{}^2 \mp \frac{(1-3t_{\sigma_1}^2)}{2\sqrt{1+9t_{\sigma_1}^2}}\lambda_4\hat{w}_2\hat{w}_s$ $-\frac{1}{2}(\lambda_5+\lambda_6)\hat{w}_s{}^2$, $\lambda_7 = -\frac{4(1-3t_{\sigma_1}^2)^2}{(1+9t_{\sigma_1}^2)^2)}(\lambda_2+\lambda_3)\frac{\hat{w}_2{}^2}{\hat{w}_s{}^2} \mp \frac{(5-3t_{\sigma_1}^2)}{2\sqrt{1+9t_{\sigma_1}^2}}\lambda_4\frac{\hat{w}_2}{\hat{w}_s}$
C-IV-a	$(\hat{w}_1 e^{i\sigma_1}\ ,\ 0\ ,\ \hat{w}_s)$	$\mu_0^2 = -\frac{1}{2}(\lambda_5+\lambda_6)\hat{w}_1{}^2 - \lambda_8\hat{w}_s{}^2$, $\mu_1^2 = -(\lambda_1+\lambda_3)\hat{w}_1{}^2 - \frac{1}{2}(\lambda_5+\lambda_6)\hat{w}_s{}^2$, $\lambda_4 = 0$, $\lambda_7 = 0$
C-IV-b	$(\hat{w}_1\ ,\ \pm i\hat{w}_2\ ,\ \hat{w}_s)$	$\mu_0^2 = (\lambda_2+\lambda_3)\frac{(\hat{w}_1{}^2-\hat{w}_2{}^2)^2}{\hat{w}_s{}^2}$ $-\frac{1}{2}(\lambda_5+\lambda_6)(\hat{w}_1{}^2+\hat{w}_2{}^2) - \lambda_8\hat{w}_s{}^2$, $\mu_1^2 = -(\lambda_1-\lambda_2)(\hat{w}_1{}^2+\hat{w}_2{}^2) - \frac{1}{2}(\lambda_5+\lambda_6)\hat{w}_s{}^2$, $\lambda_4 = 0$, $\lambda_7 = -\frac{(\hat{w}_1{}^2-\hat{w}_2{}^2)}{\hat{w}_s{}^2}(\lambda_2+\lambda_3)$
C-IV-c	$(\sqrt{1+2c_{\sigma_2}^2}\hat{w}_2\ ,\ \hat{w}_2 e^{i\sigma_2}\ ,\ \hat{w}_s)$	$\mu_0^2 = 2c_{\sigma_2}^2(1+c_{\sigma_2}^2)(\lambda_2+\lambda_3)\frac{\hat{w}_2{}^4}{\hat{w}_s{}^2}$ $-(1+c_{\sigma_2}^2)(\lambda_5+\lambda_6)\hat{w}_2{}^2 - \lambda_8\hat{w}_s{}^2$, $\mu_1^2 = -[2(1+c_{\sigma_2}^2)\lambda_1 - (2+3c_{\sigma_2}^2)\lambda_2 - c_{\sigma_2}^2\lambda_3]\ \hat{w}_2{}^2$ $-\frac{1}{2}(\lambda_5+\lambda_6)\hat{w}_s{}^2$, $\lambda_4 = -2c_{\sigma_2}(\lambda_2+\lambda_3)\frac{\hat{w}_2}{\hat{w}_s}$, $\lambda_7 = c_{\sigma_2}^2(\lambda_2+\lambda_3)\frac{\hat{w}_2{}^2}{\hat{w}_s{}^2}$
C-IV-d	$(\hat{w}_1 e^{i\sigma_1}\ ,\ \pm\hat{w}_2 e^{i\sigma_1}\ ,\ \hat{w}_s)$	$\mu_0^2 = -\frac{1}{2}(\lambda_5+\lambda_6)(\hat{w}_1{}^2+\hat{w}_2{}^2) - \lambda_8\hat{w}_s{}^2$, $\mu_1^2 = -(\lambda_1+\lambda_3)(\hat{w}_1{}^2+\hat{w}_2{}^2) - \frac{1}{2}(\lambda_5+\lambda_6)\hat{w}_s{}^2$, $\lambda_4 = 0$, $\lambda_7 = 0$

Table 5.4: The Second Conditions Of Possible Complex Vacuum Configuration For 3HDMs [97].

Tabel (5.5) will explain the final conditions of **(CVC)** and it will be as the following:

Vacuum	w_1, w_2, w_s	**Constraints**
C-IV-e	$(\sqrt{-\frac{s_{2\sigma_2}}{s_{2\sigma_1}}}\ \hat{w}_2 e^{i\sigma_1},\ \hat{w}_2 e^{i\sigma_2}\ ,\ \hat{w}_s)$	$\mu_0^2 \ = \ \frac{s^2_{2(\sigma_1-\sigma_2}}{s^2_{2\sigma_1}}(\lambda_2+\lambda_3)\frac{\hat{w}_2{}^4}{\hat{w}_s{}^2}$
		$-\frac{1}{2}(1-\frac{s_{2\sigma_2}}{2\sigma_1})(\lambda_5+\lambda_6)\hat{w}_2{}^2 - \lambda_8 \hat{w}_s{}^2\ ,$
		$\mu_1^2 \ = \ -(1-\frac{s_{2\sigma_2}}{2\sigma_1})(\lambda_1-\lambda_2)\hat{w}_2{}^2 - \frac{1}{2}(\lambda_5+\lambda_6)\hat{w}_s{}^2\ ,$
		$\lambda_4 \ = \ 0\ ,$
		$\lambda_7 \ = \ -\frac{s_{2(\sigma_1-\sigma_2)}}{s_{2\sigma_1}}(\lambda_2+\lambda_3)\frac{\hat{w}_2{}^2}{\hat{w}_s{}^2}$
C-IV-f	$(\sqrt{2+\frac{c_{(\sigma_1-2\sigma_2)}}{c_{\sigma_1}}}\hat{w}_2 e^{i\sigma_1}\ ,\ \hat{w}_2 e^{i\sigma_2}\ ,\ \hat{w}_s)$	$\mu_0^2 = -\frac{(c_{(\sigma_1-2\sigma_2)}+3c_{\sigma_1})c_{(\sigma_2-\sigma_1)}}{2c^2_{\sigma_1}}\lambda_4\frac{\hat{w}_2{}^3}{\hat{w}_s}$
		$-\frac{c_{(\sigma_1-2\sigma_2)}+3c_{\sigma_1}}{2c_{\sigma_1}}(\lambda_5+\lambda_6)\hat{w}_2{}^2 - \lambda_8\hat{w}_s{}^2\ ,$
		$\mu_1^2 \ = \ -\frac{c_{(\sigma_1-2\sigma_2)}+3c_{\sigma_1}}{c_{\sigma_1}}(\lambda_1+\lambda_3)\hat{w}_2{}^2 - \frac{1}{2}(\lambda_5+\lambda_6)\hat{w}_s{}^2$
		$-\frac{3c_{2\sigma_1}+2c_{2(\sigma_1-\sigma_2)+c_{2\sigma_2}}+4}{4c_{(\sigma_1-\sigma_2)}c_{\sigma_1}}\lambda_4\hat{w}_2\hat{w}_s\ ,$
		$\lambda_4 \ = \ -2\frac{c_{(\sigma_2-\sigma_1)}}{c_{\sigma_1}}(\lambda_2+\lambda_3)\frac{\hat{w}_2}{\hat{w}_s}\ ,$
		$\lambda_7 \ = \ -\frac{c^2_{(\sigma_2-\sigma_1)}}{c^2_{\sigma_1}}(\lambda_2+\lambda_3)\frac{\hat{w}_2{}^2}{\hat{w}_s{}^2}$
C-V	$(\hat{w}_1 e^{i\sigma_1}\ ,\ \hat{w}_2 e^{i\sigma_2}, \hat{w}_s)$	$\mu_0^2 \ = \ -\frac{1}{2}(\lambda_5+\lambda_6)(\hat{w}_1{}^2+\hat{w}_2{}^2) - \lambda_8\hat{w}_s{}^2\ ,$
		$\mu_1^2 \ = \ -(\lambda_1+\lambda_3)(\hat{w}_1{}^2+\hat{w}_2{}^2) - \frac{1}{2}(\lambda_5+\lambda_6)\hat{w}_s{}^2\ ,$
		$\lambda_2+\lambda_3 \ = \ 0\ ,$
		$\lambda_4 \ = \ 0\ ,$
		$\lambda_7 \ = \ 0$

Table 5.5: table
The Final Conditions Of Possible Complex Vacuum Configuration For 3HDMs [97].

5.5 Theoretical Constraints On Dark Matter Masses

The Standard Model is considered one of the most well-known scientific achievements in physics, especially in particle physics, because it contains the basic building blocks for the rest of the physical particles. Despite these achievements, it could not explain the existence of Dark Matter particles, in addition to not explaining their masses. It is possible to search for the existence of extended physical models and theories that would give a scientific explanation for the true masses of Dark Matter particles, knowing the masses of Dark Matter is considered one of the important physical topics in the department of particle physics, which will certainly give a wonderful progress in revealing important physical phenomena in the near future. Many modern scientific experiments are trying to reach how to calculate the masses of Dark Matter particles, among these scientific researches are those produced by the **ATLAS Detector** at the **Large Hadron Collider (LHC)**. The **(LHC)** is considered the vital physical largest accelerator for elementary particles in the world with the highest energies, as this accelerator has great merit in detecting most of the Standard Model particles, the most important of which is the Higgs Boson. The **(LHC)** consists of important **nine** detectors that play a pivotal role in detecting unknown physical particles, as these detectors will be mentioned as follows [98]:

- ATLAS: A Toroidal LHC Apparatus.
- CMS: Compact Muon Solenoid.
- LHCb: LHC-beauty.
- ALICE: A Large Ion Collider Experiment.
- TOTEM: Total Cross Section, Elastic Scattering and Diffraction Dissociation.
- LHCf: LHC-forward.
- MoEDAL: Monopole and Exotics Detector At the LHC.
- FASER: ForwArd Search ExpeRiment.
- SND: Scattering and Neutrino Detector.

The ATLAS Detector is considered the main detector in the process of accelerating and colliding physical particles, in addition to being the most important of the nine physical detectors that make up the Large Hadron Collide **(LHC)**. The ATLAS Detector is a very large cylindrical detector with a mass of **7000** tons, a length of **46** meters and a diameter of **25** meters [99]. One of the important physical topics of the ATLAS Detector is that it, along with another vital detector of the **(LHC)** detectors called **Compact Muon Solenoid (CMS)**, conducted a very important experiment that led to the detection of the Higgs Boson in **2012**. The ATLAS Detector is very specialized in detecting unknown physical particles such as Dark Matter particles and Supersymmetric particles [100]. The importance of the ATLAS Detector in the Large Hadron Collider **(LHC)** is unparalleled as it is the most important physical detector in it, as it bears the greatest responsibility in carrying out scientific experiments that would lead to the detection of the presence of unknown physical particles in addition to providing the necessary practical explanations for these experiments. As a result, it was necessary to address the parts of the ATLAS Detector in order to understand how it works in the **(LHC)**. The basic components in the ATLAS Detector work in an integrated manner with each other, as each component has its own mission. These components will be mentioned as follows [101]:

- Muon Detectors and Solenoid.
- Electromagnetic Calorimeters and Forward Calorimeters.
- End Cap Toroid and Barrel Toroid.
- Inner Detector and Hadronic Calorimeters.
- Shielding

Figure (5.1) shows all component of ATLAS Detector in the Large Hadron Collider.

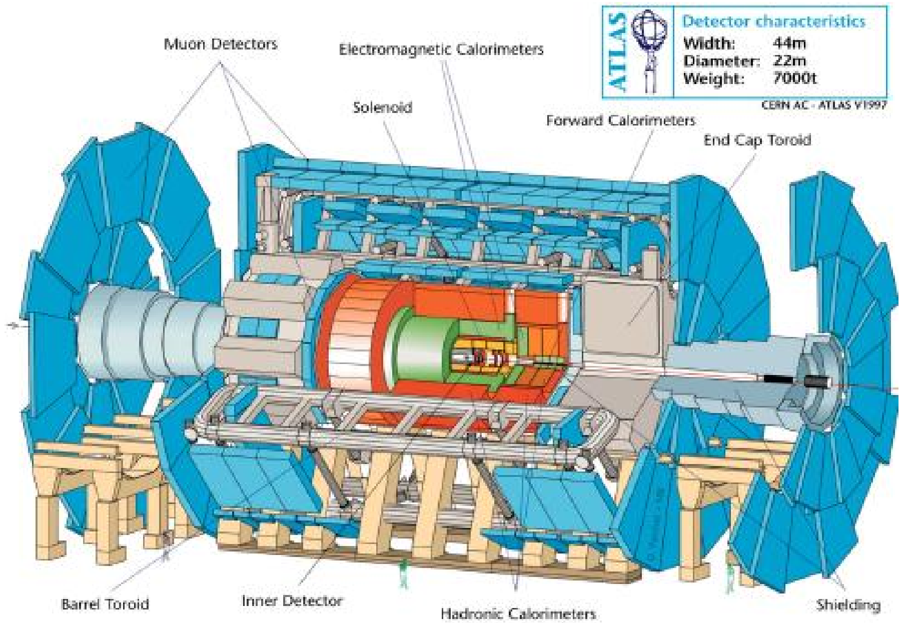

Figure 5.1: The ATLAS Detector [100].

The researches of the ATLAS Detector located in the Large Hadron Collider **(LHC)** focuses in particular on unknown physical phenomena for which no scientific explanation has been provided, for example Dark Matter, its particles and masses, which has led to an increase in scientific incentives among particle physicists in order to start doing many experiments. Modern physicists in the **(LHC)**, especially in the ATLAS Detector, could obtain accurate scientific explanations for these physical phenomena. The topic of determining the masses of Dark Matter is a very important physical topic, its importance lies in proving that Dark Matter has physical particles like the rest of the Standard Model particles whose masses are known, this would allow the world of physics, especially particle physics, to get much closer to the truth about Dark Matter particles and learn more about their possible candidates. As a result of this topic, many important scientific physical researches that resulted from the ATLAS Detector will be addressed, which are directly related to Dark Matter Masses.

It is known that the Standard Model has achieved amazing scientific achievements in the world of particle physics, but it is considered among the low energy levels, but this is no longer useful with the emergence of all of the new degrees of freedom and the symmetries of modern physical systems. This led to the start of relying on very high energy levels in the **(LHC)**, especially in the ATLAS Detector, in order to reach accurate scientific explanations regarding unknown physical phenomena such as dark matter, its particles and masses [102]. In **April** of the year **2021**, a scientific research was published by the ATLAS Detector, in which it talks about the ranges of Dark Matter masses by relying on an Active High Energy Photons in a series of Proton-Proton collisions used in the ATLAS Detector at the **(LHC)**. The main idea of this paper is that Proton-Proton collisions will reveal the masses of the Dark Matter particles candidates by stimulating a High-Energy Photons with certain large transverse momentum ranges as well as using a center of mass-energy of about **13 TeV** **(**$\sqrt{s} = 13\ Tev$**)**, which also requires masses of quarks, leptons and vector bosons, all of that is in order for the interactions between all these particles to occur through proton-proton collisions within high energy ranges [103].

The working mechanism of this research in order to calculate the masses of Dark Matter Candidates is that the **Dirac Fermion Dark Matter Candidates** will interact with the quarks within a series of interactions that will be emitting photons. These photons, which form the basis of how the masses of these candidates are calculated, will be associated with Axion particles through certain coupling constants from each of the quarks, leptons and Dark Matter particles. As a result, new chains of reactions will occur within certain ranges of the high energy needed for that [104]. After breaking the electroweak symmetry of the interactions between photons and axions, the coupling constants of quarks, leptons and Dark Matter particles will become more effective and will be transformed into free operators that have a significant impact on the Lagrangian equation of photons and Axions. This would help in calculating the masses of the required Dark Matter particle candidates. Finally, based on the calculations of this scientific paper by the ATLAS Detector, the mass ranges of Dark Matter particle candidates range from **415 Gev/c²** to **580 Gev/c²** [105].

The previous research that resulting from the ATLAS Detector at the Large Hadron Collider **(LHC)** is considered one of the good scientific researches in the history of particle physics. It provided many important physical explanations about the ranges of masses of Dark Matter particle candidates that have been of interest to particle physics for decades to the present day. Scientific researches on finding Dark Matter masses are still ongoing, which leads to getting very close to the correct actual values for the masses of Dark Matter particles and not just giving ranges of masses for these particles.

5.6 Summary

Finally, the end of this chapter has been reached and a detailed discussion of a very rich model extended to the Standard Model, namely (3HDMs), was being discussed. The necessary equation of its potential was also addressed and written in terms of new basis for physical fields in order to facilitate the work of the required mathematical calculations. This transformation was through a very special transformation matrix that led to the transformation from an old basis to a new basis and then write the special equations for each new basis. Tables of **Real Vacuum Configurations (RVC)** and tables of **Complex Vacuum Configurations (CVC)**, which form the basic basis of this model, were also addressed by setting appropriate constraints and finding derivation equations to find the necessary relationships that link the coupling constants (μ, λ), this would determine the mathematical equations linking the masses of particles that are expected to be Dark Matter Candidates and then the possibility of linking these equations in order to obtain correct mass ranges for these particles. The subject of the **Large Hadron Collider (LHC)** was also discussed and its nine detectors were briefly identified, but its main detector, which is called the ATLAS Detector, had the largest share of explanation and interpretation because of its very high importance in the **(LHC)**. The ATLAS Detector was also identified and all its parts were identified, in addition to that an important advanced scientific research was clarified in the ATLAS Detector related to the ranges of Dark Matter masses, where this research was explained in a careful useful brief way. **In the next chapter**, only the Real Vacuum Configurations will be addressed, and in particular the case **(R-I-1)** will be carefully studied, where the equations for the coupling constants (μ, λ) will also be presented after substituting the constraint values for this case $(w_1, w_2, w_s) = (0, 0, w_s)$. After that, the masses equations of particles that will represent the Dark Matter Candidates will be derived. This requires the presence of the necessary **Software micrOMEGA** to calculate the relic density and the necessary table (6.1) will be drawn for that. Finally, the mass ranges of these particles will be determined, and thus the required results will be obtained for this thesis.

Chapter 6

The R-I-1 Vacuum Configuration

In this chapter, we will study the Real Vacuum Configuration **(R-I-1)** with vacuum expectation values that will be shown as follows:

$$(w_1, w_2, w_s) \;=\; (0, 0, w_s) \tag{6.1}$$

This configuration has many interesting properties such as:

1. All parameters in the potential are real and the potential is CP-conserving.

2. It Preserves the S_3 symmetry spontaneously, where the symmetry $h_1 \to -h_1$ is already present in the potential as we discussed in the previous chapter and the expectation values for this configuration are $< h_1 > = < h_2 > = 0$ and $< h_s > = w_s$.

3. The field h_s is the only active field in this case, meaning it acts similar to the Standard Model Higgs field and give masses to other matter particles (Fermions and Gauge Bosons). While the two fields h_1 and h_2 are non active or inert because they have zero vacuum expectation values and do not interact with ordinary matter.

4. The Dark Matter Particles reside in the inert Higgs fields h_1 and h_2.

5. When studying the physical spectrum, the (R-I-1) configuration results in zero number of massless scalar particles, or Goldstone Bosons.

We substitute the values

$$(w_1, w_2, w_s) = (0, 0, w_s)$$

in the minimization equations Eq. (5.32) and Eq. (5.33) to get:

$$\mu_0^2 \;=\; -\lambda_8 w_s^2 \tag{6.2}$$

$$\mu_1^2 \;=\; -\frac{w_s^2}{2}(\lambda_5 + \lambda_6 + 2\lambda_7) \tag{6.3}$$

Where again, we restrict our study on real parameters only in the potential.

6.1 Higgs Basis Transformation and Physical Spectrum

The **Physical Spectrum** - mainly the masses of observed particles - are invariant and model independent. This fact makes it convenient to make the transformation and work in the so-called Higgs Basis where the fields are denoted by (H_1, H_2, H_3). The relation between the Higss Basis Fields and the fields (h_1, h_2, h_s) are given via a rotation matrix that has two phases β_1 and β_2.

For the case under study, (R-I-1), the phases β_1 and β_2 are defined as:

$$\tan\beta_1 = \frac{w_s}{w_1}$$

and

$$\tan\beta_2 = \frac{w_s}{w_2}$$

when substituting

$$w_1 = w_2 = 0$$

we get:

$$\beta_1 = \beta_2 = \frac{\pi}{2}$$

And the transformation is written on the matrix form as:

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_s \end{pmatrix} \tag{6.4}$$

We see that the H_1 field transforms as the h_s field and this field H_1 is the equivalent of the Standard Model Higgs field, i.e. it is the active field which will give masses to the ordinary matter particles. The other two Fields H_2 and H_3 transforms as h_1 and h_2 up to a minus sign and these two fields will be non active or inert and will produce the inert scalar particles which accommodate Dark Matter.

The total number of scalar Higgs-like particles that result from this model are **9** particles. One of them is the Standard Model Higgs Boson and we denote it by the symbol H_1^0. The other **8** particles which are basically inert particles and have **Mass Degeneracy** among themselves as it is clear from the rotation matrix above. The mass degeneracy comes in the scalar Higgs-like particles: **2 neutral pairs** that we denote by $(H_2^0, H_3^0, A_2^0, A_3^0)$ and **2 charged pairs** that we denote by $(h_2^\pm, h_3^\pm)$. This means that we have only **3** different masses in the inert part in this configuration. The masses of these inert particles are given in terms of the potential parameters as follows:

$$m^2_{H_2^0} = m^2_{H_3^0} = \mu_1^2 + \frac{w_s^2}{2}(\lambda_5 + \lambda_6 + 2\lambda_7) \tag{6.5}$$

$$m^2_{A_2^0} = m^2_{A_3^0} = \mu_1^2 + \frac{w_s^2}{2}(\lambda_5 + \lambda_6 - 2\lambda_7) \tag{6.6}$$

$$m^2_{h_2^\pm} = m^2_{h_3^\pm} = \frac{1}{4}(2\mu_1^2 + w_s^2\lambda_5) \tag{6.7}$$

The **lightest** of the neutral particles is the Dark Matter candidate since it does not decay to other heavier scalar particles due to simple **Mass Conservation Law**, this means it is stable. Also, it is neutral which the basic properties of Dark Matter Particles.

6.2 Mass Order of Inert Higgs Particles

We chose the lightest scalar particle to be the H_2^0 or equivalently, H_3^0 because of the mass degeneracy as explained above. In fact, the order of the masses of theses scalar particles is:

$$m^2_{A^0_{2,3}} > m^2_{h^{\pm}_{2,3}} > m^2_{H^0_{2,3}} \tag{6.8}$$

Of course, one could rearrange the parameters to chose the neutral inert scalar particle A_2^0 or equivalently A_3^0 to be the lightest neutral particle and be the Dark Matter Candidate. This choice does not change anything in the conclusion about this case since the parameters are related as seen in Eqs. (6.5) to (6.7) above.

6.3 Numerical Analysis For R-I-1 Vacuum Configuration

We perform numerical analysis for the vacuum configuration **(R-I-1)** which is quite CPU extensive since we deal with many parameters some of them are independent and others are dependent. Also, we apply several checks on the data points to see whether it fits the basic theoretical constraints on the potential such as the stability of the scalar potential. We fixed the value of the mass for the Standard Model Higgs Boson to be: $m_{H_1^0} = \mathbf{125.25}$ GeV which was taken from the Particle Data Group Webpage. We also used the other **8** inert masses as input parameters (These are only **3** parameters due to mass degeneracy). For the **Relic Density** evaluation in this model, we use the **Software micrOMEGAs**. We consider the value for the relic density from this model (3HDMs) that falls in the following range of Cold Dark Matter Relic Density ($\Omega_{CDM}h^2$) is being acceptable.

$$\Omega_{CDM}h^2 = 0.12 \pm 0.01 \tag{6.9}$$

Note: This is a relaxed range of relic density for cold dark matter compared to what is provided by the Particle Data Group.

Benchmark Point	$m_{H_2^0}$	$m_{H_3^0}$	$m_{h_2^\pm}$	$m_{h_3^\pm}$	$m_{A_2^0}$	$m_{A_3^0}$	μ_1	λ_a	Ωh^2
A	80.1	80.1	95.76	95.76	128.26	128.26	78.4	0.0089	0.112
B	54.3	54.3	85.32	85.32	112.71	112.71	48.61	0.0191	0.113

Table 6.1: Selected values of benchmark points for the configuration (R-I-1). All masses are in GeV. The mass $m_{H_1^0}$ is fixed to the Standard Model Higgs Boson of 125.25 GeV.

In the Table (6.1), we chose to present two benchmark points for this (R-I-1) configuration: Point **A** with the lightest neutral Inert Higgs has a mass of about **80 GeV** and Point **B** where the lightest neutral Inert Higgs has a mass of about **54 GeV**. Both points amounted to an acceptable relic density value of about **0.11**. The symbol λ_a in the Tabel (6.1) will be as follows:

$$\lambda_a = \lambda_5 + \lambda_6 + 2\lambda_7 \tag{6.10}$$

Chapter 7

Conclusion

In this thesis, we started by a review of the properties of Dark Matter and evidences that make scientists believe that Dark Matter exist. We also reviewed the Standard Model which has one doublet of the Higgs Field that is active and responsible of giving the ordinary matter particles their masses. The Standard Model is known to fail for accommodating Dark Matter. We then reviewed a simple extension of the Standard Model called the (IDM) which is a version of the Two Higgs Doublet Models (2HDMs) with one of the Higgs Doublets being inert and produce inert particles that accommodate Dark Matter. We then studied a S_3 - symmetry for Three Higgs Doublet Model (3HDMs), then we studied analytically and numerically a specific vacuum configuration of this model called (R-I-1), which allows for two doublets to be inert and one doublet to be active. The Dark Matter particles which are scalar, neutral and light, reside in the inert doublets H_2 and H_3 in the Higgs Basis notation. After performing numerical analysis using the **Software micrOMEGA**, we found that the inert scalar particles of this configuration are massive enough to account for the Dark Matter as the relic density we got from our numerical results lies within the acceptable range for relic density calculated mainly in [21]. **Finally**, the (3HDMs) are very rich models for investigating Dark Matter Particles. The numerical and analytical analysis of these models are very complicated and **Central Processing Unit (CPU)** consuming given the fact that there are many parameters to deal with. Also, for future work, one could investigate other vacuum configurations that are either real or complex and might lead to CP violation. Also, we can study Dark Matter particles predicted by other important theories such as the **String Theory** (see Appendix **A**).

Appendix A

String Theory For Dark Matter Candidates

A.1 The Weakly Interacting Massive Particle

The Weakly Interacting Massive Particle (WIMP) are Subatomic, Heavy, Electromagnetic Neutral Particles that do not interact with the electromagnetic field, they were hypothesized to be the main component of Dark Matter [106]. It was believed that these particles (WIMP) are heavy (not light) and move at a slow speed (not close to the speed of light), because if the Dark Matter particles were light and fast moving, these particles would not gather in the density fluctuations that led to the formation of galaxies in this universe [107]. This thing was previously explained in section (2.3) when we talked about the Lambda Cold Ddrk Matter Model, where the word cold denotes that the Dark Matter particles are heavy and slow in motion, meaning that their speed does not approach the speed of light [108]. It was also assumed that (WIMP) are not baryonic particles, (**Baryon Particles are the particles that consist of the union of three quarks with each other such as protons and neutrons**). The reason for not being baryonic particles is that the abundance of baryonic particles in the universe is a high percentage after the Big Bang, in addition to that, the exact nature of these particles (WIMP) was not revealed until now [109].

A.2 The String Theory

The String Theory considered a vital theory in particle physics that predicted the existence of other particles other than those in the Standard Model such as **Dark Matter Candidates (Axion, Axino, Gravitino)**, in addition to predicting important particles that named (WIMP) [110]. This theory worked to explain physical phenomena that the Standard Model could not explain, as the Standard Model was unable to explain both the mass of neutrinos and the particles of Dark Matter [111]. Therefore, it was necessary to discover the String Theory which helped in solving these problems and providing special explanations for each of them [112]. String Theory talked about the possibility of weakly interacting particles at the electro weak level that could explain Dark Matter, where many experimental tests were conducted which would provide many necessary explanations regarding Dark Matter and its particles, this thing is one of the necessary theoretical constraints to know the extent of the reality of Dark Matter [113]. Many assumptions related to Dark Matter Particle's have been able to assume that these particles are (WIMP), in addition to that, some experiments have proven that these particles have little interaction **(Dark)**, in addition to their speed far from the speed of light **(massive particles)** [114].

The Research and experiments are still ongoing in order to know the reality of (WIMP), which would help discover the origin of Dark Matter, this help to identify the properties of Dark Matter Particle's, so that these particles can enter the Standard Model and thus be part of it like other particles [115]. The main idea of these research's that works to detect weakly interacting massive particles is to find the largest possible theoretical space (finding the largest possible number of scientific models) necessary to reveal the truth of Dark Matter [116], this includes the possibility of increasing the strength of interactions from the weak interaction to a strong interaction for (WIMP), working to increase the strength of interactions from weak to strong is a necessary matter, according to what researches related to (WIMP) have talked about, but there is something that must be taken into consideration, which is to increase the range of the masses of (WIMP). All of this in order to reach the structure of these particles and then access to the ideal interpretation of Dark Matter [117]. It was assumed that mass particles with little interaction are a candidate for the formation of Dark Matter Particle's, but other hypotheses talked about the possibility of the presence of a **Dark Sector** that contributes to revealing the presence of other stable particles that may be very weak in interaction or be light that would give an explanation about the components of Dark Matter, these particles are **Axion**, **Axino** and **Gravitino** that stemmed from the String Theory [118]. Since these particles are weakly interacting, this means that they have little interaction between them or little interaction with other particles, this thing may give important explanations about how Dark Matter particle's interact or how they behave with particles of other materials or how to be part of the particles that found in the Standard Model and other explanations can be obtained [119].

A.3 Dark Matter Candidates - The Axion

The Axion Particle is an elementary particle that has been postulated in order to solve a very important problem in particle physics, which is the **Strong Charge Parity Problem (SCPP)**. This problem is especially present in the physics of Quantum Chromo Dynamics **(QCD)** [120], where the **(QCD)** relates to the interpretation of the charges of elementary particles found in the Standard Model, in addition to interpreting the colors issued by them in their own interactions [121]. There is an important symmetry that required in **(SCPP)** and is also required in all physics subjects, especially particle physics, which is the symmetry **(CP)**, where it divided into two parts, charge symmetry and spatial coordinates symmetry [122]. The **(CP)** symmetry states that the laws of physics must be the same if there is an exchange between the particle and its anti particle, this is known as symmetry **(C)** while the symmetry in the spatial coordinates that only reversed, this is known as symmetry **(P)** [123]. It constitutes an important role in explaining weak interactions, how they occur, factors affecting them and their relationship to strong interactions. All of these things must be taken into account in order to study them correctly and thus obtain better results for these interpretations and studies. The **(CP)** symmetry is also explain the relationship between any particle and its anti particle [124]. The **(QCD)** is concerned with explaining the interactions of particles in the Standard Model, as well as explaining new particles that could be among the particles of the Standard Model, such as the Axion. With regard to this particle and its relationship with this theory, the effects of **(QCD)** may produce effective potentials that the **Axion Field** can move, and this would cause fluctuations of this field around the **Minimum Effective Potential (MEP)** in order to reach the stability process for the Axion [125].

Many recent experiments have been done on the Axion particle in order to prove that it has mass or not. The results of these experiments said that this particle has a mass of about **ten times or slightly more** than the mass of the electron particle, where it is expected that the Axion particle is one of the Dark Matter Candidate's [126]. Despite the tremendous success achieved by the Standard Model in describing elementary particles as well as explaining how these particles interact, there are many problems that Standard Model has not been able to solve them, including the Strong Charge Parity Problem [127].

There are several important physical symmetries related to the Axion, the most important called **Peccei - Quinn Symmetry**. This symmetry will be broken dynamically by the structure of the vacuum and also by many influencing factors that help break it, this would produce an Axion particle through a series of reactions resulting from this break, in addition to providing a **Non Perturbative Axion Gluon Coupling** [128]. This coupling has very important effects that help in obtaining some explanations related to Peccei - Quinn symmetry that would provide a solution to the problem of Strong Charge Parity. It provide a small mass for the Axion, and the reason for this is the association of the Pion particle with the Axion. The mass of the Axion ($\mathbf{M_a}$) will be approximated by the following equation [129]:

$$\begin{aligned} M_a =& \frac{f_\pi m_\pi}{4 < \Phi_a >} \sqrt{\frac{4 m_u m_d}{(m_u + m_d)^2}} \ [\ 1 + O(m_{u,d}/m_s)\ ] \\ &\approx\ 0.6\ \times\ 10^{-3}\ eV \left[\frac{10^{10} Gev}{f_a}\right] \end{aligned} \tag{A.1}$$

- m_u : Mass of Up Quark.
- m_d : Mass of Down Quark.
- m_s : Mass of Strange Quark.
- m_π : Mass of Pion Particle.
- f_π : Pion Decay Constant.
- f_a : Axion Decay Constant.
- Φ_a : Axion Field.

The experimental constraints and researches on the Axion particle are still continuing to the present time, as many scientists described the process of searching and detecting the Axion as a rather strict process, due to the lack of ease in this topic. This requires great effort and hard work to obtain the best explanations and then reach to the best results about the Axion [130]. The mass of the Axion was reviewed in equation (A.1), it was found that this mass is very very small (**Less than 0.01 eV**), therefore, this thing increases the difficulty of detecting Axion mass through the experiments. As a result, there is a need for **Astrophysics Constraints** that concerned with the physics of the universe, especially in the stars [131]. These constraints indicate that the Axion will be invisible particle due to the difficulty of detecting it until now and also its has a very small mass [132].

It is possible to infer the Axion through the emission of neutrons during the process of cooling stars, which is one of the stages of formation of the stars [133]. All these explanations and observations would give a value for the **Decay Constant** of the Axion ($\mathbf{f_a}$), it's greater than or equal to $\mathbf{(10^9)}$ **Gev**. This value with respect to the Peccei and Quinn scale will be approaches to the following value [134]:

$$f_a = 3 \times 10^{10}\ Gev \tag{A.2}$$

A.4 Dark Matter Candidates - The Axino and The Gravitino

The Axino is a hypothetical particle that has been postulated by many important theories in particle physics, especially the String Theory. It is expected that the Axino is one of the lightest elements that are described as **Super Symmetric**. This description makes it one of the Dark Matter Candidate's [135–137].

The Gravitino is also a hypothetical particle like the Axino particle, which was postulated by the theory of **Super gravity** [138, 139], this theory combines the properties of both General Relativity and String Theory. It is expected that the gravitino particle is one of the Dark Matter Candidate's. In addition, it is considered a super symmetric fermion partner of the **Graviton** particle. It is known that the fermions have a spin of half odd integers such as $(\frac{1}{2})$ or $(\frac{3}{2})$ .... etc. but for the gravitino its spin is equal to value of $(\frac{3}{2})$. It follows the **Pauli Exclusion Principle** which states that no two or more fermions can have the same four quantum numbers because of the difference in the Spin Quantum Number for each of these fermions. These quantum numbers will be in the following [140–142]:

- The Principal Quantum Number.
- The Sub Quantum Number.
- The Magnetic Quantum Number.
- The Spin Quantum Number.

As for the Gravitino, it has own characteristics that will be explained as follows:

1. The Gravitino is in a **Stable State**. This means that it may be the lightest of the super symmetric elements, this would make it one of the Candidates of Dark Matter Particle's and thus constitute a percentage of the Density of Dark Matter such as the Axion and the Axino [143–145].

2. The Gravitino is in an **Unstable State**. This means that it will decay only during certain interactions called Gravitational Interactions. So that, it is not expected in this case to contribute to the Density of the Dark Matter. Therefore, it will has a half life period and also has a Decay Constant. This is known as the Gravitino Cosmological Problem [143–145].

A.5 The Decay Of Dark Matter Candidates

In general, the process of decay occurs for the elements that are in a **State of Instability**, as they decay into other elements in order to reach the **State of Stability**. The process of decay is not a simple process where most of the decay processes need a long series of necessary reactions. The occurrence of the decay process for the Candidates of Dark Matter Particles (Axion, Axino, Gravitino) is an important matter, this process is considered one of the most important theoretical and experimental constraints on the existence of Dark Matter in this universe, this thing would help in revealing more about the reality of this Dark Matter and identifying their particles [146]. The idea of the subject of decay for Dark Matter Candidate's is necessary and its especially important for both the Axino and the Gravitino. The two particles are **Closely Inter Connected** to each other, this thing would led to the discovery of very important equations where these equations will play a necessary role in linking each of the following matters: **Decay Constants** , **Masses** and **Half Life Times**. For both Axino and Gravitino, there are two possible states of decay to each other based on their masses. These two states will be explained as follows [147]:

1. The Gravitino Particle is **heavier** than the Axino Particle. In other words, its mass is the greatest. This would lead to the decay of the Gravitino to the Axino within a certain decay chain reaction. In this case the Decay Width of the Gravitino will be given by the following equation [148, 149]:

$$\Gamma_{3/2} = \frac{m_{3/2}^3}{192\pi\ M_P^2}(1-R)^2(1-R^2)^3 \tag{A.3}$$

- $\Gamma_{3/2}$: Decay Width For Gravitino Particle.
- $m_{3/2}$: Mass Of Gravitino Particle.
- M_P : Planck Scale.
- R : Mass Ratio between Axino and Gravitino.
- R = $m_{\tilde{a}}\ /\ m_{3/2}$.

2. The Axino Particle is **heavier** than the Gravitino Particle. In other words, its mass is the greatest. This would lead to the decay of the Axino to the Gravitino within a certain decay chain reaction. In this case the Decay Width of the Axino Particle will be given by the following equation [150, 151]:

$$\Gamma_{\tilde{a}} = \frac{m_{\tilde{a}}^5}{96\pi\ m_{3/2}^2\ M_P^2}(1-R^{-1})^2(1-R^{-2})^3 \tag{A.4}$$

- $\Gamma_{\tilde{a}}$: Decay Width For Axino Particle.
- $m_{3/2}$: Mass Of Gravitino Particle.
- $m_{\tilde{a}}$: Mass Of Axino Particle.
- M_P : Planck Scale.
- R : Mass Ratio between Axino and Gravitino.
- R = $m_{\tilde{a}}$ / $m_{3/2}$.

It is known that there is a relationship between the **Half Life Time** and the **Decay Constant** or the **Decay Width**. This thing is very necessary in identifying the relationship between the masses of the decaying particles and the time required for their decay. Concerning the candidates of Dark Matter Particles, especially the decay of both Gravitino and Axino Particles. The following equations shows the correlation of each of the masses of these decaying particles with the time required for decay. These equations will be explained as follows [152, 153]:

$$\tau_{3/2} = \frac{In(2)}{\Gamma_{3/2}} \tag{A.5}$$

$$\tau_{3/2} = \frac{In(2)\ 192\pi\ M_P^2}{m_{3/2}^3\ (1-R)^2(1-R^2)^3} \tag{A.6}$$

$$\tau_{\tilde{a}} = \frac{In(2)}{\Gamma_{\tilde{a}}} \tag{A.7}$$

$$\tau_{\tilde{a}} = \frac{In(2)\ 96\pi\ m_{3/2}^2\ M_P^2}{m_{\tilde{a}}^5\ (1-R^{-1})^2(1-R^{-2})^3} \tag{A.8}$$

- $\tau_{3/2}$: Half Life Time For Gravitino Particle.
- $\tau_{\tilde{a}}$: Half Life Time For Axino Particle.

There are three important conditions of Mass Ratio between Gravitino and Axino **(R)** that will be explained as follows [154]:

- **The First Condition $\mathbf{R \longrightarrow 0}$**

In this case, the mass of the Gravitino is much **more greater** than the mass of the Axino ($\mathbf{m_{3/2} >> m_{\bar{a}}}$). When we substitute the value of (**R**) into equations (A.6 , A.8), it will be concluded that the half life time of the Gravitino will depend only on its **Mass**. While the half Life time of the Axino will approach zero. Therefor, the new form of equations (A.6 , A.8) will be as follows:

$$\tau_{3/2} = \frac{In(2)\ 192\pi\ M_P^2}{m_{3/2}^3} \tag{A.9}$$

$$\tau_{\bar{a}} \longrightarrow 0 \tag{A.10}$$

- **The Second Condition $\mathbf{R \longrightarrow 1}$**

In this case, the mass of the Gravitino is **approximately equal** to the mass of the Axino ($\mathbf{m_{3/2} \approx m_{\bar{a}}}$), and therefore the value of (**R**) goes to the value (**1**). When we substitute the value of (**R**) into these two equations (A.6 , A.8), the value of the half life time of each of these particles will approach to the infinity. So, it will be difficult to reach the reality of the decay of both particles and thus also the difficulty in explaining them as candidates of Dark Matter. In this case, each mass of both particles will not be dependent on the half life time of each of them. The half life time of both of them will go to the infinity as the following:

$$\tau_{3/2} \longrightarrow \infty \tag{A.11}$$

$$\tau_{\bar{a}} \longrightarrow \infty \tag{A.12}$$

- **The Third Condition $\mathbf{R} \longrightarrow \infty$**

In this case, the mass of the Gravitino will approach to **Zero** ($\mathbf{m_{3/2} \approx 0}$). In other words, the mass of Axino is much **more greater** than the mass of Gravitino. This would make the value of (**R**) approach to the infinity. When we substitute the value of mass ratio (**R**) in each of the two equations (A.6 , A.8), the half life time of the Gravitino will approach to zero while the half life time of the Axino will depend on both its mass and the mass of the Gravitino as follows:

$$\tau_{3/2} \longrightarrow 0 \tag{A.13}$$

$$\tau_{\bar{a}} = \frac{In(2)\ 96\pi\ m_{3/2}^2\ M_P^2}{m_{\bar{a}}^5} \tag{A.14}$$

A.6 Possibility of Entering Dark Matter Candidates into the Standard Model

- After reviewing the decay of Dark Matter Candidate's, in particular the Axino particle and the Gravitino particle, the important question will now be asked, which is: **Is it possible for Dark Matter and its particles to exist within the Standard Model and be part of it in the near future ???**

Until this moment, the existence of particles of Dark Matter has not been detected, although the experiments and the research are still continuing in the process of careful research, in addition to the work of many important theories, in order to reach a correct understanding of Dark Matter and its particles. There is a possibility in the near future to detect particles of Dark Matter, and thus the possibility of introducing them to the Standard Model, as is the case with all the particles that make up the Standard Model. There are many ideas issued by some physicists regarding the reality of Dark Matter particle's, these ideas are very important as they help in obtaining even a very simple hope for identifying Dark Matter and its components, some of these ideas and opinions issued by these scientists will be presented as follows:

1. **Peter Voke, Ph.D. Physics, London (1976).**

He talked about the Standard Model, where he said that there is an extension of the Standard Model known as the **Super Symmetric Minimum Extension**. This extension can include Dark Matter particle's in the future, meaning that Dark Matter can consist of the lightest Super Symmetric Particles [155].

2. **Ian Kimber,Physics,UCL,1965, FRAS MIET MinstP LRPS EMI CRL 1967-99.**

He talked about gravity, as he said that it is a weak force that is not enough to detect dark matter, but the only way to detect it is at the largest galactic scales. He also talked about strong and weak interactions, as he considered these interactions to be the basis for forming the nuclei of short-lived atoms and not just relying on the force of gravity, and therefore it is illogical to say that particles interact only through gravity [155].

3. **Art Hobson, Prof of Physics, Univ. of Arkansas, 1964-present (1964-present).**

He said that it is possible in the future for Dark Matter particle's to enter the Standard Model and be part of it, such as Fermions and Bosons. He also said that there is a lot of laboratory research aimed at revealing the reality of the Dark Matter particle's, as these things are not particles and therefore should be called **Quanta**, until it is actually discovered and then it must be given the name (Particle) [155].

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